VOLUME LV NUMBER 1 WHOLE 480

SCHOOL SCIENCE AND MATHEMATICS

JANUARY 1955

School Science and Mathematics

A Journal for All Science and Mathematics Teachers

All matter for publication, including books for review, should be addressed to the editor. Payments and all matter relating to subscriptions, change of address, etc. should be sent to the business manager.

Entered as second class matter December 8, 1932, at Menasha, Wisconsin, under the Act of March 3, 1879. Published Monthly except July, August and September at 450 Ahnaip St., Menasha, Wis. PRICE. Four dollars and fifty cents a year; foreign countries $5.00; current single copies 75 cents.

Contents of previous issues may be found in the Educational Index to Periodicals.

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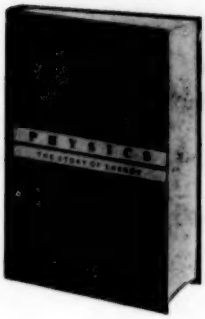

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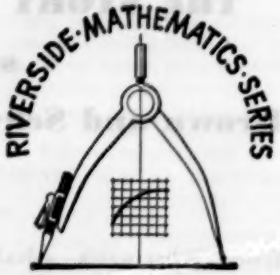

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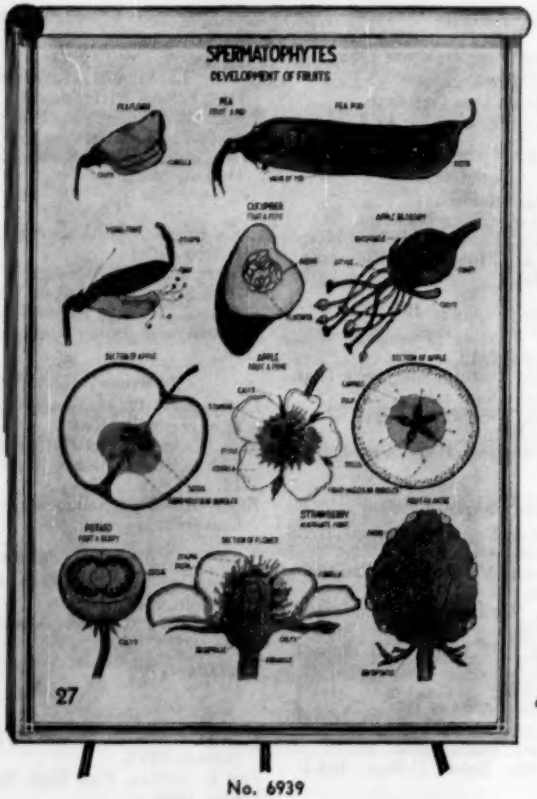

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THE ATCHAFALAYA: PRINCE OR PIRATE?

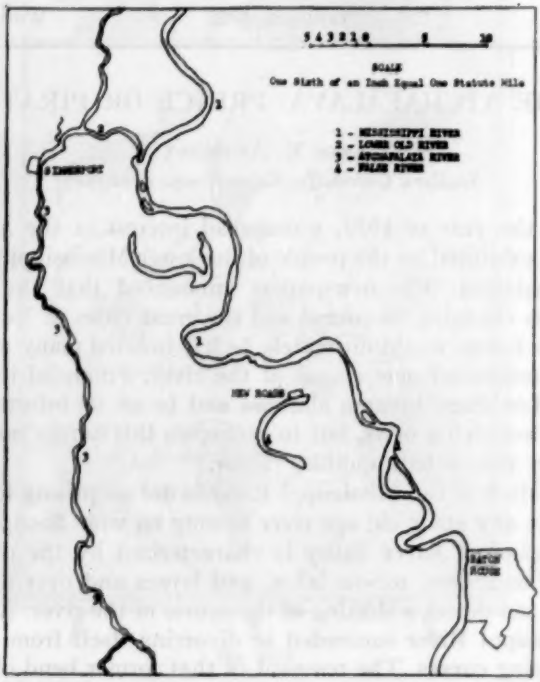
JAMES K. ANTHONY
Southern University, Baton Rouge, Louisiana

During the year of 1953, widespread interest in the Mississippi River was exhibited by the people of the lower Mississippi River valley of Louisiana. The newspapers announced that the *Father of Waters* was changing its course and the great cities of New Orleans and Baton Rouge would ultimately be left isolated many miles away from the estimated new course of the river. Financial interests of southern Louisiana became alarmed and to an un-informed public this was electrifying news, but to geologists this hardly rated a comment other than a half audible, "Hmm!"

The conduct of the Mississippi River is not surprising for it is behaving like any other old age river flowing on wide floodplains. The lower Mississippi River valley is characterized by the presence of numerous meanders, oxbow lakes, and levees and over a period of years one can detect a shifting of the course of the river. About 1722 the Mississippi River succeeded in divorcing itself from one of its many looping curves. The remnant of that former bend of the river is now known as False River on whose western bank is located the city of New Roads. This crescent-shaped lake is one of several and the former route of the Mississippi River can easily be traced by connecting these oxbow lakes to the present course of the river. The above is common to all rivers that are designated *old age*. An explanation of the stages of a river may serve to clarify this problem.

A river is termed *youthful* when it flows in a narrow V-shaped channel or valley; has few tributaries; is still cutting downward into rocks; and transports a mixture of alluvium from coarse particles to fine silt. Rivers in upland regions are inevitably in the youthful stage and

the Colorado River of the Grand Canyon region is a classic example. Rivers are *mature* when they flow in broad valleys with the beginning of floodplains; meanders are in evidence; maximum number of tributaries; and prominent deposition of sand and silt. The lower Ohio River exhibits this characteristic. Some of the many characteristics of an *old age* river are numerous meanders and oxbow lakes (meanders that are cut off from the main channel of the river); wide floodplains; sand and silt deposited in the river channel (sand bars); lateral deposition of sediment (levees); deposition of sediment at the mouth of the river (delta); and an infrequent changing of the channel.

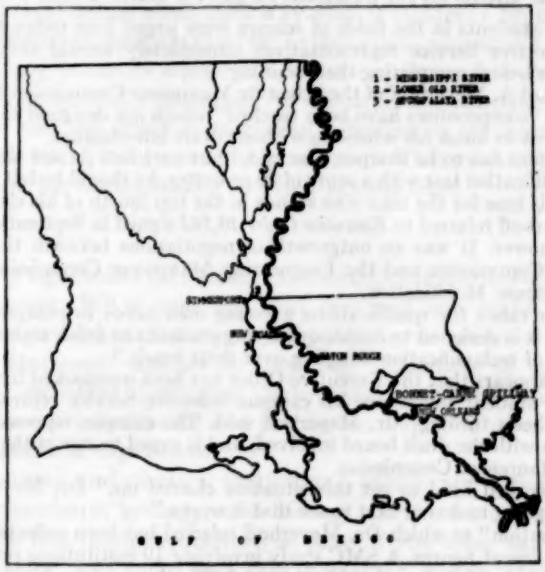


MAP 1

At the northern boundary of Pointe Coupee Parish is a small river called Lower Old River (Map 1). It flows in an arc westward from the Mississippi River past the city of Simmesport to the Atchafalaya River and is an accomplice in the stream piracy of the Atchafalaya. If this piracy of the Atchafalaya goes unchecked then it is estimated that in about twenty-five years the Mississippi River will flow southward to the Lower Old River and continue on to the Atchafalaya and thence to the Gulf of Mexico. The Atchafalaya flows through

rich agricultural land of southern Louisiana. In fact, much of this land is cut-up by the many tributaries and bayous of the Atchafalaya and with an increase in the volume of water from the Mississippi even a greater amount of this land (given over to the growing of sugar cane, soybeans, yams, cotton, and hot peppers) will become marshes and swamps. This might mean more space for Louisiana's teeming wildlife.

The cities of Baton Rouge and New Orleans on the other hand would cease to maintain their importance as major Gulf Coast shipping ports of petroleum products and industrial and agricultural



MAP 2

commodities. Already the *New Orleans States** has reported that the salt water of the Gulf of Mexico is in evidence in the Mississippi River and is a threat to the water supply of New Orleans (Map 2).

However, all is not lost nor is the future of the lower Mississippi River a futile one. With pending funds from the state and federal coffers, the Lower Old River will be sufficiently dammed to prevent any more of its piratical activities. The United States Corps of Engineers, who have made thorough and exhaustive studies of this problem, will cut off the flow of water to the Atchafalaya. If the Mississippi River rises and reaches flood stage, then sluices will be opened and the surplus water will be allowed to flow into the Lower Old

* *New Orleans States*, November 13, 1953.

River, much the same way that the Bonnet-Carre spillway just north of New Orleans conducts excessive water from the Mississippi River to Lake Pontchartrain.

Thus, what started out to be a serious menace to an industrial area becomes just another challenge to man in overcoming one of Nature's wiles. And man, now in possession of the facts, can go back to his interest in this mad whirl of living and, like the geologists, utter a half audible, "Hmm!"

SCIENCE GRADUATE STUDENTS HAVE NEW DRAFT RIGHTS

Graduate students in the fields of science were urged here today to see their campus Selective Service representatives immediately should they get their draft notices before completing their training.

Dr. Howard A. Meyerhoff of the Scientific Manpower Commission told *Science Service* that "compromises have been reached" which are designed to enable the better student to finish his schooling without draft interruption.

"The student has to be sharper," he said, "but once he's passed that Selective Service qualification test with a score of 80 or better, he should be left alone. The same thing is true for the man who stands in the top fourth of his class."

Dr. Meyerhoff referred to *Executive Order 10,562* signed in September by President Eisenhower. It was an outgrowth of negotiations between the Scientific Manpower Commission and the Engineering Manpower Commission with the Office of Defense Mobilization.

The order raises the qualifications a young man needs to postpone military service, but it is designed to enable qualifying students to finish training without the "worry of reclassification dangling over their heads."

When it appears that the Executive Order has been overlooked by local draft boards, the student should see his campus Selective Service representative to start the wheels turning, Dr. Meyerhoff said. The campus representative will get in touch with the draft board involved, and is urged to report the case to the Scientific Manpower Commission.

"We've worked hard to get this situation cleared up," Dr. Meyerhoff said, "and we're going to do our best to see that it works."

The "situation" to which Dr. Meyerhoff referred has been reflected in recent college enrollment figures. A SMC study involving 19 institutions revealed that between May and October of 1953, at least 2,000 advanced graduate students in science and engineering "had been denied the opportunity to complete their studies." Dr. Meyerhoff said the situation still exists.

When asked whether students were using advanced studies as an academic draft-dodging technique, Dr. Meyerhoff said that he has no evidence of this.

"There isn't one in 20 who tries for graduate work to evade the draft," he said. "In the first place, they don't evade the draft. They merely delay it.

"And it's more of a sacrifice to stick to school until training is complete than to enter service at a younger age. But certain scientific and engineering training must be continuous, or else you lose men. Their interest may be diverted in the meantime, or they may forget the complex mathematics necessary in the field."

Packaging tape for food stores permits bananas, celery and lettuce heads to be bundled quickly for counter display. Made in red, green, blue or yellow in ¾- and ½-inch widths, the 60-yard rolls of paper tape can be marked on with pencil, pen or crayon. The tape can be used also to seal cellophane and plastic bags.

MATHEMATICAL IDEAS

JOSEPH MAYER*
Miami University, Oxford, Ohio

The study of mathematics after the elementary grades are passed frequently continues in disappointment. This is not because any great amount of technique needs to be learned before the discipline can be effectively used. There is nothing esoteric in the procedure of letting a letter, like "*x*," stand for an unknown quantity. The mystery (if one appears) arises when such a symbol is suddenly brought forth without adequate preparation or explanation, especially before a class of young students whose minds are unprepared to grasp spontaneously the symbol's meaning. An off-hand introduction to a new symbolism has the effect of fastening a feeling of strangeness upon the subject, which may never be eradicated. This probably explains the aversion for mathematics so often encountered.

I

The most significant fact about mathematics is its over-all generalizing character. When you say that one and one make two, you generalize—you do not limit yourself to apples or horses or anything else. The statement applies to *anything*, regardless of size, color, sentiments, or other qualifying circumstances. All particular characteristics are brushed aside and you fasten attention upon the most general aspects of experience.

In astronomy and physics, motion and energy are important concepts; in chemistry, geology, and biology, matter is; in psychology, the mind is. But in mathematics you may, if you wish, take motion, energy, matter, and mind and call them four *things*. Mathematics deals with things as things—*any* thing and *every* thing. It is only as you move from mathematics up the scale to the other sciences that you begin to differentiate between things; first you distinguish the categories of motion, energy, matter, and mind, and then more limited categories. To be sure, all the sciences are engaged upon this generalizing quest, "to discern the general in the particular and the permanent in the transitory," but the greatest generalizer of them all is mathematics.

II

The simplest and most obvious generalization that can be made about things pertains to number. If you have had occasion to put

* The author, one-time Instructor in Mathematics, University of Texas, is at present head of the Department of Economics at Miami University and is the author of "The Seven Seals of Science."

your "things" into storage, you doubtless made a note of the number of pieces. There were fifty-two, let us say, and such a statement appears to represent a simple enough mental process. Yet there is already a considerable amount of mathematical development wrapped up in the generalization "fifty-two." There are tribes living today who find it difficult to count beyond twenty or even ten and who are forced to use fingers and toes as aids in the operation of counting.

The idea of " numbering" developed early in human experience, although our so-called Arabic notation, with its simple and convenient symbolism, is not many centuries old. The Greeks used a notation similar to the complicated Roman numerals still inscribed on public buildings today (MDCCCCXXV, e.g., for 1925), and other races employed a still more complicated symbolism.[1]

The process of numbering involves two basic mental operations; one indicating a summation, as in the statement that I have fifty-two things in storage; the other indicating relative order or position, as in the statement that the favorite horse came in third. The "ordinal" concept opens up the subject of series of numbers. Pythagoras noticed that the sum of a certain ordinal series, starting with unity and taking every other number, always forms a perfect square. Thus, $1+3=4$ or 2^2; $1+3+5=9$ or 3^2; $1+3+5+7=16$ or 4^2, etc. It is possibly because of this peculiarity that we have come to call such numbers "odd." Other early pioneers arranged numbers in different orders or sequences and worked out some of the principles of permutations and combinations. A further study of never-ending or "infinite" series brought to light additional laws underlying such questions as convergence and divergence; also, the exceedingly practical concepts of approximations and limits, e.g., a regular polygon with "n" sides approximating a circle as the number of sides (n) is increased.

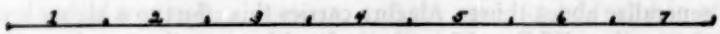
From earliest antiquity the concept of whole or integral numbers had been established, i.e., one, two, three, etc., which in the familiar Hindu-Arabic symbols have come to be written, 1, 2, 3, etc. As early as the Egyptian and Babylonian periods the idea of fractional numbers, $\frac{1}{2}$, $\frac{1}{3}$, had taken root, the utilization of fractions being associated with such practical matters as dividing an estate between two or more claimants.

The Greeks, with their ability for viewing ideas geometrically,

[1] Addition and subtraction were difficult enough to the Romans and Greeks, while only specially trained computers could multiply and divide. Archimedes and Euclid would have been dumbfounded at the facility with which the modern grammar school child handles problems in long division and multiplication.

The essential "place-value" feature of the Hindu-Arabic notation, however, was used by the Babylonians as early as 1600 B.C., if not before. In other respects, also, the Babylonians exhibited an exceedingly high level of mathematical attainment. (See O. Neugebauer, "The Exact Sciences in Antiquity," Princeton University Press, 1952, pp. 5, 14, 17–22, 28, 47).

used lines, triangles, squares, etc. to represent number concepts and thus deduced some of the more important properties of numbers. Integers, for example, they represented as equal divisions of a straight line, thus:[2]



By further subdivision, fractional numbers were readily pictured, in which connection the interesting fact was noted that a fraction, like $\frac{2}{3}$, may be looked upon either as a subdivision of the line representing "one" or as a ratio of one line to another, as of the line "two" to the line "three." Thus developed the concepts of ratios and proportions, destined to play an important role in later scientific thinking.

Additional study of ratios and series led to other discoveries of number properties of considerable consequence, as the incommensurables like $\sqrt{2}$, those in which no exact fraction of the original length of line exists.[3]

Through a further examination of the properties of a straight line as made up of a series of integers, fractions, and incommensurable numbers, mathematicians were ultimately led to wonder if such a series actually composes the whole of a straight line. Might there not be gaps in the line not filled by any numbers—fractional, incommensurable, or other? Hence arose the ideas of continuity and discontinuity.

In generalizing about things, the major concept of a real positive number is undoubtedly among the first to enter human consciousness. Closely associated with positive numbers, however, are negative numbers. The antithesis of present and absent, alive and dead, things seen and then missed—these are indissolubly linked in human experience. It is not merely a question of the positive presence of a person over against nothing when he is gone, but over against an absence which is more than a neutral non-existence. In our sophisticated experience it is easy to find additional examples in every-day life—an overdrawn account, "in the hole" in a game of cards, a temperature below zero.

Along with the concepts of positive and negative must also have arisen in earliest antiquity a notion of the meaning of zero, even though our present symbols for showing these three ideas did not come into general use until fairly recent centuries. The wide acceptance of other compact modern symbols such as $>$, $<$, $\sqrt{\ }$, $=$, $(\ )^n$, $\div$, $\times$, was likewise relatively recent, dating probably from around the time of the discovery of America.

[2] The Greeks, of course, used their own symbols for numbers.

[3] Incommensurables are also called irrationals or surds. This suggests that originally these concepts must have been regarded as "without reason" or "absurd."

III

The rules of adding, subtracting, multiplying, and dividing numbers, which are comprehended in that division of mathematics known as arithmetic, represent the first systematic advance in man's effort to generalize about things. Algebra carries this effort to a higher level of abstraction. While arithmetic deals with particular numbers, i.e., 5, 91, 163, algebra deals with numbers in still more general terms. Certain properties of a particular number, e.g. that twelve can be divided by 2, 3, 4, and 6, do not apply to all numbers; other properties like addition and subtraction do. It is of value to separate the more general properties of numbered things from more particular properties. Thus in algebra we start with the statement, Let "a" or "z" be any number, and we examine what follows. The results are summed up in the laws of algebra, from the simple statement that $a+z=z+a$ through the more complicated quadratic solution,

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a},$$

derived from the general statement, $az^2+bz+c=0$, with which students become familiar in high school.

It has been found useful in the solution of problems to differentiate between numbers that are known and those that are unknown but whose values are sought. Conventionally we use the first letters of the alphabet to designate known quantities and the last letters to represent the unknowns. Hence a, b, c etc., have come to stand for *any known* numbers and z, y, x, etc., for *any unknown* ones. Another mere matter of convenience but of great value in mathematical reasoning.

The significance, in simplifying mental processes, of allowing a symbol like "x" to stand for an unknown quantity, is undoubtedly generally appreciated today. If I am given a problem to solve, for example to find a number such that the sum of its half, third, and ninth part is equal to thirty-four, the mental operation would be somewhat difficult if I did not immediately say: Let "x" be the number sought; then

$$\frac{x}{2}+\frac{x}{3}+\frac{x}{9}=34;$$

and by placing the left-hand side of the equation over the common denominator eighteen, it follows that

$$\frac{17x}{18}=34 \quad \text{or} \quad x=36,$$

which is the answer. This problem is so simple that it can be solved also by arithmetical means, or by a Joel Kupperman in his head; but, even so, the symbol "x" is an economizer of thought processes. For the solution of more difficult problems its use is indispensable.

From crude beginnings, the various algebraic laws of addition, subtraction, multiplication, division, and the rest—to be found in any standard textbook today—gradually took form; yet in the process of deriving them a perplexing difficulty arose. Integral, fractional, incommensurable, positive, and negative numbers did not seem to comprehend the whole gamut of the needs of full generality in algebra. A very disconcerting kind of "number" was constantly being encountered, or, rather, the algebraic laws deduced could not be considered as having full applicability unless a meaning could be attached to what was long thought an impossible concept. This finally received the designation of the "imaginary." In contradistinction, all other number concepts—those considered in preceding paragraphs—are called "real."

To illustrate the difficulty, take a simple algebraic equation such as $x^2-a=0$. Solving, $x^2=a$ and $x=\pm\sqrt{a}$. Testing this equation to see if it satisfies all known values for "a," it is seen at a glance that when "a" is positive no difficulty arises. If a negative value such as "-1" be given to it, however, x becomes equal to $\pm\sqrt{-1}$, which corresponds to no conceivable real number. Finally, after considerable confused philosophizing, $\sqrt{-1}$ came to be accepted and the symbol "i" (later "j" also) was assigned to represent it. Still the question of the validity of the concept continued to be raised until, in relatively recent times, a way of representing the idea graphically was provided.

One additional development of importance should be mentioned here. This has to do with the most general algebraic form employed today. Although symbols for zero had come to be utilized very early, its significance with respect to general algebraic form was not worked out until much later. A long train of important consequences, such as the ability to formulate general equations like the quadratic, flows from the convention of equating a given combination of mathematical symbols to zero. This we now achieve by the simple device of collecting all the quantities or terms except zero on the left hand side of the equality sign, thus leaving zero alone on the right, such as was illustrated above in the general form of the quadratic. More complicated equations of higher powers may frequently be simplified or broken down into quadratics or cubics by arranging them first in the form of zero on the right and all other terms on the left. This is again, of course, wholly conventional, but it is nevertheless of first importance today in the solution of algebraic problems.

IV

Side by side with the analysis of things regarded as numbers, whether particular or general, there developed another major mathematical generalization about things, viz., their space relations, meaning their relative positions and their sizes and shapes. Such considerations belong largely to the field of geometry. Here the Greeks made their greatest mathematical contribution. It was their visualizing power, in addition to their superb logical faculties, which did so much to transform the crude materials developed in still earlier times into a structure worthy to be called a science.

The fundamental concept at present regarding the space relations of things is to view them as points situated "here" or "there," the hereness and thereness being indicated by an origin and a position, as in the following example:

$$\begin{array}{cccccc} & & & & & P \\ \dot{0} & \dot{1} & \dot{2} & \dot{3} & \dot{4} & \dot{5} \end{array}$$

The point "P" is here regarded as being five units distant from the origin "0" horizontally to the right. From this elementary concept many important geometrical generalizations flow. Significant advances in these respects, however did not come until comparatively late, the Greek accomplishments having to do chiefly with the sizes and shapes of objects rather than with their positions in space.

Natural objects are found in an endless variety of forms and sizes. In its search for generality, geometry attempts to pick out the most representative and universal, the "perfect" ones, of which all others are modifications. It had been found that size, like color, was a modifying circumstance which could be set aside so far as the general purposes of geometry are concerned. Regarding shape, the triangle, the square, the hexagon, and other regular polygons; the circle, the ellipse, and other "conic sections"; were among the plane figures that the ancient Greeks analyzed. Similarly with the regular solid figures. The general properties of these figures, such as their areas or volumes, were determined in considerable detail. Yet this was only a beginning, for behind the concept of shape lay the still more fundamental concept mentioned above, viz., that of a point in its motion generating a line, the line generating a plane figure, and the plane figure generating a solid.

The Greek mathematical genius showed itself at its best in the invention of plane conic sections. By 300 A. D. the structure of such figures (secured by cutting a conical "dunce's cap" with planes at various angles) had been completed; and from that time until the 17th century, the subject was studied largely as an abstract science

unrelated to the practical affairs of the world. Then Kepler utilized the principles of the ellipse to unlock the mysteries of the movements of the planets in their courses.

Trigonometry had a practical origin associated mainly with surveying. By calculating and tabulating the fixed ratios between the sides of right angle triangles for the various degrees of the circle, a very important labor-saving tool was developed. Later it, too, was seen to have a wide application to natural phenomena, especially to astronomy and to the unceasing ebb and flow of periodic or cyclical events surrounding us on every hand.[4]

V

The reduction of geometrical concepts to their lowest elements in terms of stationary and moving points began somewhat prior to the days of René Descartes in the 17th century, but it was not until his epoch-making invention of coordinate geometry that the more fundamental ideas received wide acceptance. By relating plane geometrical curves and figures to a system of coordinates—to two axes, one horizontal and one vertical let us say, and a so-called origin at their point of intersection (from which plotted curves are measured and to which the curves are at every point referred)—Descartes focused attention upon the moving point as a fundamental concept. In addition to reducing geometric form to its lowest terms, coordinate geometry developed the concepts of "here" and "there" into a science. Rectangular coordinates are not the only systems of reference that can be used and that are used today, but the principles remain the same.

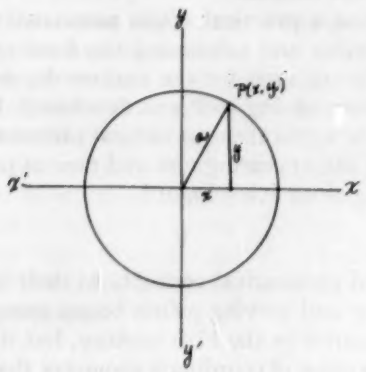
Cartesianism brought together the hitherto separate fields of algebra and geometry and made them one. Now not only could the locus of a point in its movement be viewed as forming a geometrical figure, but it could also be regarded as representing an algebraic equation. Henceforth a circle could be represented by the general equation $x^2+y^2=a^2$, and this equation could in turn be thought of as the circle, in the figure on the following page. Similarly with other mathematical equations and figures. The continued quest for general laws with respect to things had finally resulted in working out the principles

[4] In historical perspective, geometry and trigonometry have had a much longer and wider employment than the arithmetic and algebra that are current today. The latter, in fact, did not come into general use until recent centuries. This situation is well expressed by F. Sherwood Taylor in his "Short History of Science and Scientific Thought" (Norton, 1949, pp. 80–81), as follows:

"Algebra had been introduced by the Arabs, but in 1500 most of what we would do by algebra was done by geometrical methods. The successive powers of numbers x, x^2, x^3, were thought of as lines, areas, and solids; and so the notion of x^4 was as difficult to them as the fourth dimension to us, because they could not imagine it geometrically. . . .

"In the fifæenth century ordinary people worked out sums with an abacus or counters, or by clumsy methods using Roman figures; and *algorism*, as working in Arabic figures was called, was still a learned accomplishment. In the sixteenth century educated people learnt to use Arabic figures and the multiplication table, though a good many knew it only to five-times. The labor of astronomical calculations was enormous, and only at the beginning of the seventeenth century was it eased by the invention of logarithms."

of number, form, and position and in relating them intimately to one another.



VI

Epoch-making as was the invention of Descartes, it paved the way for still more significant developments in the next generation, in the rise of the calculus. Thus far, in analyzing the most general properties of things, we have dealt primarily with things at rest. The numbers, positions, and forms of things in a static state has been the main theme. Now a whole new department of mathematical study is opened up, viz., the study of things in motion. The concept of a moving point not only became basic for the older departments of mathematics, but it marked the beginning of entirely new ones. It was largely with the new tool of the calculus that Sir Isaac Newton established physics as a science.

It is not possible in a short article to develop the meaning of the calculus and its analysis of change and rates of change, involving such matters as variables, functions, infinitesimals, and limits. For a detailed understanding of these and related ideas the reader must be referred to a standard text on the subject.

VII

There have been several broad periods of mathematical advance: first, the Greek period (preceded by Egyptian and especially by Babylonian beginnings) in which certain foundations were laid; second, the medieval Hindu-Arabic period which gave us our system of numbers and saw substantial improvements in algebraic technique; third, the period of the 16th and 17th centuries which witnessed the rise of coordinate geometry and the calculus, the simplification of mathematical symbolism, the invention of logarithms as an aid to com-

putation, and the further development of trigonometry and algebra; fourth, the period of the 18th century in which the calculus and its derivatives were extensively applied to practical problems in astronomy and physics; and, finally, the fifth or recent period beginning with the 19th century.

The past century and a half has witnessed mathematical progress in three major directions: one, in a further development of geometry, algebra, and the calculus; two, in a critical examination of the foundations of pure mathematics; and three, in important advances in applied mathematics. Limited space forbids our pursuing these developments here. A broad outline, however, may be secured from the author's "Seven Seals of Science," pp. 216–225.[5]

The significance of mathematical reasoning as a whole was well expressed by Ernst Mach, a noted physicist, from whom we quote in closing:

"Thought-economy is most highly developed in mathematics, that science which has reached the highest formal development, and on which natural science so frequently calls for assistance. Strange as it may seem, the strength of mathematics lies in the avoidance of all unnecessary thoughts, in the utmost economy of thought-operations. The symbols of order, which we call numbers, form already a system of wonderful simplicity and economy. When in the multiplication of a number with several digits we employ the multiplication table and thus make use of previously accomplished results rather than repeat them each time, when by the use of tables of logarithms we avoid new numerical calculations by replacing them with others long since performed, when we employ determinants instead of carrying through from the beginning the solution of a system of equations, when we decompose new integral expressions into others that are familiar, —we see in all this but a faint reflection of the intellectual activity of a Lagrange or Cauchy, who with the keen discernment of a military commander marshals a whole troop of completed operations in the execution of a new one."

[5] See also, Richard Courant and Herbert Robbins, "What is Mathematics?," Oxford University Press, 1941, pp. xv–xix.

ENGLISH SCIENTIST AT WISCONSIN

To understand human destiny "we need to understand as much as possible about nature, about man, and about man's place in nature," Julian Huxley, famed English scientist, told an audience at the University of Wisconsin.

This is the first time in history in which man has arrived at a "reasonably comprehensive and reasonably scientific and true picture of his destiny, thanks to the extension of knowledge in the fields of evolution, psychology, sociology, history, and archeology," Huxley said.

He contended that man's development to date appears to be the summit of the evolutionary process on earth "and that his role in nature is now seen as being the agent or instrument for further evolutionary advance."

Man's aim is now the realization of new possibilities for himself and greater fulfilment, individually and as a group.

"Before attempting this task, we will have to sit down to a big job of study and discussion and education to decide what are the most desirable fulfilments for which we should strive, and what are the best ways of realizing them in practice," Huxley said.

SCIENCE DAY

An Annual Event at Western Michigan College

More than 1000 eager and enthusiastic science students with their teachers from 36 different high schools in southwestern Michigan gathered on the campus of Western Michigan College on April 9, 1954, to participate in the events of our Fifth Annual Science Day. Science Day at our college has become an institution designed to stimulate the interest of promising students in science and mathematics.

Eight of our college departments (agriculture, biology, chemistry, geography and geology, mathematics, paper technology, physics and psychology) cooperate actively in arranging and presenting our Annual Science Day program. The Administrative Staff of the College, the staff of our College Radio Station, and the college services of publicity, audio-visual aids, and research and testing always render very active assistance and make invaluable contributions to the success of our Science Day programs. Our student newspaper, *The Western Herald*, publishes a special Science Day edition, which is distributed free to the visiting high school students.

The events of Science Day fall into three main categories:

1. *A competitive comprehensive scholarship examination in science and mathematics. The winner of this examination is awarded a $150 scholarship at Western.* The runner-up is awarded a $100 scholarship. These scholarships may be renewed annually by the recipient for the three remaining years of his undergraduate study, provided he continues his work in science and mathematics and maintains a good college record. Twenty-seven high school science students entered this competitive examination in 1954.
2. *A convocation science lecture of a popular nature planned at a senior high school level.* Dr. Warren J. McGonnagle, Research Physicist of the Argonne National Laboratory, gave the 1954 lecture-demonstration on the topic, "Everyday Uses of the Atom."
3. *Exhibits, displays, demonstration experiments, and motion pictures* presented in classrooms and laboratories by the cooperating college departments. College students in science and mathematics participate extensively in planning, preparing, and presenting these activities. Among the more popular features in the departmental presentation in 1954 were:

   Agriculture: Some practical aspects of scientific agriculture.

   Biology: Blood typing of about 250 visiting students, color motion pictures "Beaver Valley" and "Nature's Half-acre," shown continuously throughout the day to about fifteen hundred visitors and college students.

   Chemistry: A semi-humorous skit entitled "All Wet," consisting of a series of spectacular experiments; continuous motion pictures related to chemical processes; a number of student-operated experiments, including pH measurements, fire extinguishers and water softeners.

   Geography and Geology: A rock clinic, to which visiting high school students were invited to bring rocks for identification.

   Mathematics: An exhibit of navigation and surveying instruments; "Olympic Contest" tests in mathematics used in USSR, both in the original Russian and in English translation; a motion picture on the "Isograph," a device for finding imaginary roots of real polynomial equations.

   Paper Technology: Actual making, coloring, and testing of paper.

   Physics: Planetarium, amateur radio station in operation, television camera transmitting a picture, cloud chamber, scintillation and geiger counters, numerous student experiments.

   Psychology: Display and demonstrations of apparatus and devices for psychological testing, including a modified lie detector.

Gerald Osborn
Walter G. Marburger
Western Michigan College
Kalamazoo, Michigan

ALL IN ONE DAY

MILDRED MARIE HARMAN
Northern Garrett County Jr.-Sr. High School, Accident, Maryland

CHARACTERS

Jane	Mother
John	Father
Sue	Their Daughter
Junior	Their Son

SCENE I

Narr. The first scene takes place in the kitchen of John Smith's farm home. It is a nice, warm enchanting morning in late May. Everything is quiet in the early morning except for an occasional clatter and bang of pans as Jane, John's pretty wife goes about the task of preparing breakfast. We find her now making coffee.

Jane: John likes his coffee made with a tablespoon of coffee to a cup of water. (Measures coffee) One, Two. (She goes to the sink and measures two cups of water.) One, Two. There! It's ready for the stove. (Sets coffee on the stove, steps back and takes a good look at her new electric stove.) It sure is nice not to have to build fire in the morning. Electricity is so much faster and cleaner, too. (Looks lovingly at the stove) Why, these lines remind me of some of the geometry I had in high school! I have never realized before, how much math was connected with a stove. For example: (touching each in turn) this temperature scale for the oven, the minute and second timer, and the automatic oven timer that turns the oven off at the right time. None of them would be possible without math. If fact, the stove would not be possible. Math is a factor of electricity and of all our modernized equipment today.

Well, it's something to think about but that won't get breakfast ready. I guess I'll have grapefruit juice. (She goes to the refrigerator and takes out a quart jar containing juice and pours half of it into four small glasses and then, surveying the remainder says:) That will be enough for one more time provided John or one of the kids doesn't find it in the refrigerator. (Jane puts a skillet on the stove and puts some butter in to melt.) Now let's see, John will eat three eggs, Junior two, Sue one and myself one. That's seven. Gee! That's math, too, It's about time for the family to come down. (She starts putting things on the table.)

(John enters looking very sleepy.)

Jane: Good morning, John.

John: What's so good about it? I didn't get enough sleep last night.

Jane: It's not my fault! You'll learn not to stay up reading half the night. Are the children coming?

John: I guess.

(Sue and Junior enter.)

Jr.: Mornin', Mom and Dad.

Sue: Good Morning!

Jane: Hello children. Breakfast is ready.

(The family sits down at the table.)

Jr.: What are you going to do today, Dad?

John: Oh, I think I'll mow some hay in the south field on the other side of the road. The weather forecast says fair this weekend.

Jr.: How much are you going to cut down?

John: Since tomorrow's Saturday we should be able to take in about six loads in the afternoon, and the hay's pretty good this year. How much do you think, will be in an acre?

Jr.: Oh, I'd say about $2\frac{1}{2}$ loads to the acre.

John: Yes, I guess that's about right. I better mow somewhere around two and one half acres then.

Jr.: Say Dad, that was a math problem wasn't it? We have an assignment in math to watch for a place that mathematics turns up in our daily life. That's the example I can take.

Jane: You know, I was just looking at our new stove this morning and thinking how much math was connected with it. You would be surprised. Do you know what we should do? We should all be real mathematics minded today and see just how much math we can find around us and then report our findings at the supper table tonight.

Jr.: O. K.

John: Sounds all right to me.

Sue: Where am I going to find math?

Jane: Don't worry too much about it, Sue. You can just tell us what you had in arithmetic class and maybe you can see something else. If not though, we'll see if we can help you think of something. Now run along and get ready for school. You just have an hour to get dressed and catch the bus, you know.

(The children start off stage.)

Jr.: Now right there was a math problem because Mother had to subtract the time it is now from the time when the bus comes.

Sue: Oh, I see. When Mother tells us to hurry and get dressed, in hopes that we will still have time to do the dishes before the bus comes, that's math.

Scene II

Narr. Scene II takes place in the evening of the same day. There is an air of impatience about the house. Each member of the family is anxious to tell his experiences of the day and each hopes the others haven't forgotten to watch for signs of math.

(Jane and Sue come on stage and begin to put supper on the table. John and Junior enter and wash their hands and faces. Then they all sit down and bow their heads for a few seconds. They begin to fill their plates.)

Jane: Well, I hope you all have a better appreciation of math now than you did this morning. How shall we tell about our findings Will it be O. K. to begin with the oldest and go to the youngest?

Jr.: Sure.

Sue: Go ahead Dad.

John: O. K. The first thing I remember that I did mathematical was right after I stepped into the milk house. I started running the water to wash the cows and the very first thing I had to do was measure the detergent that the milk company suggested I use.

Jr.: Well that shouldn't have been hard, Dad.

John: It wasn't as easy as you think. The directions said use one half tablespoon for every gallon of water. As I had a 3 gallon bucket I had to use one and one half tablespoonfuls.

Jr.: What else mathematical did you do while you were milking?

John: Well, after I began milking, I decided to keep track of how much milk Star gave today. I weighed it this morning and again this evening and then I had to add the two weighings together. Also I figure the approximate number of gallons would be about $7\frac{1}{2}$ for the whole day.

Jr.: Did you do any of the mowing today?

John: Yes, I did, Junior, but when I went out I remembered that we hadn't figured how many rounds would be in $2\frac{1}{2}$ acres. I just took a guess and came up with about 12 rounds. You know, Junior all our figures on that hay are just estimates. We'll see tomorrow how good we are. At eleven-thirty I had only two more rounds to go and only a half hour to do it in. Would you have thought I could do it? Well, it just took a little math. I had been on the job since 9 o'clock. That meant I had been working $2\frac{1}{2}$ hours and had finished 10 rounds. That told me I was doing 4 rounds an hour. With 2 rounds and a half hour I decided I would be able to do it.

Sue: What happened after dinner, Dad?

John: Right after dinner a man came and wanted to buy 28 bushels

of wheat. This problem took a pencil and paper but it wasn't hard. I guess I'll tell you about one more problem concerning math before I stop.

Jane: Was it that letter you received today?

John: Yes, it was a notice which said that all stock holders in the Southern States Cooperative were to receive a dividend of $5\frac{1}{2}$ cents a share. My 385 shares are beginning to bring in more money all the time. I should be getting a check in a few days. It isn't a terrific amount but every drop in the bucket counts. Maybe we should each have something we want out of it. Be thinking what you would like to have for about five dollars.

Jr.: I would like to have a horn for my bicycle.

Sue: I want a new doll.

Jane: I'd like to have a new hat.

John: I just said be thinking, Remember?

Sue: I guess it's your turn, Mother.

Jane: All right. Here goes. When Dad was talking about weighing his milk, I wondered if you children realized the importance of scales. Nowadays it seems everything is weighed out in packages for us at the store and the only time we use the scale is when we weigh ourselves. Weighing is a very important way of measuring. It may be useful to have some knowledge of this different way of using mathematics which we cannot be without. There are other measuring units besides pounds and ounces, you know. With a system of grams you can weigh very small things. Then too, did you realize that when Dad orders a ton of feed he is ordering it by weight? I imagine you know how many pounds there are in a ton.

Sue: Sure, 2,000.

Jr.: Yes, in a short ton but in a long ton there is 2,240.

Sue: A long ton! What's that?

Jr.: Well, it used to be used a lot when people bought coal and hay by the long ton. Some places they still use it for coal. It's just another measurement. See?

Sue: I guess so.

John: How about letting mother go on.

Jane: All right. Here's some of the math that I saw today. The first thing I did when I woke up was try to think what day it was. After Friday came to my mind I wondered what the date was. And I'll tell you something. When I tried to remember that, my mind just went blank and I couldn't think of anything. So I got up and looked at the calendar. Since it was Friday, I looked down Friday's column. It wasn't the 3rd and it

wasn't the 10th and not the 17th, but I couldn't remember having a 24th so that must be what today is.

Jr.: But Mother, does the day of the year have anything to do with mathematics?

Jane: Just hold your shirt on and I'll tell you. Did you ever wonder how we got the calendar? Don't you imagine that it made a lot of work for somebody to figure a system out such as that. Just think of all the math it took coupled with quite a lot of astronomy to make a calendar that would work. And the one we have now isn't perfect. Now for some of the other problems I ran into. I made a batch of cookies today and the recipe said it only made one dozen cookies. Do you know what I did?

Sue: Made four recipes, I hope. (hopefully)

Jane: No, (laughingly) I figured 2 dozen would be enough for us at one time. Too many cookies aren't good for you, you know. Then I made a rice pudding and I only had half enough rice, so I had to cut all the other ingredients down to half, too. (stops and eats)

Jr.: Anything else Mom?

Jane: Yes, but you know, I believe we're forgetting the geometry side of our project. Look at the design in the tablecloth, the lineoleum on the floor, the rug on the living room floor, the curtains or just anything you want and you can see signs of geometry. You have it around you everywhere, all the time. Since I seem to be the treasurer of the family, or maybe you would call me just the bookkeeper, I had some problems along that line today. When Dad sold that wheat I had to record it. And when I went to town for my groceries it created quite an arithmetic problem. I got 3 loaves of bread at 17 cents a loaf, a jar of salad oil for 43 cents, 2 dozen oranges at 47 cents a dozen. A cake of yeast for 7 cents, and a bag of flour for $1.25. In addition to having to check the grocer's figures, I had to record it all in the record book. Boy, just wait till the end of the month, will I have a job!

Jane: I suppose I better stop and give Junior a chance. He looks so eager. What in the world have you been taking notes for, Jr.?

Jr.: Do you mind if I keep you in suspense about the notes awhile and tell you some of my other experiences first? I'll lead up to it by and by.

Jane: Well-ll I guess not if it's part of your story.

Jr.: O. K., then I'll begin by telling you what math I saw this morning. I had core the first three periods and we are studying

transportation. I got to thinking how much math went into the construction of our present transportation system and how dependent we are on transportation in turn for bringing clothing and especially food to us. Also, as our teacher brought out to our committee, that we wouldn't have our new school if it were not for transportation. There would be a much smaller, frame school house with one or two windows. In addition to Math helping our new school along indirectly through transportation it played a very direct part in the building of our school, too. First, the blue print has to be drawn. Then the cost had to be figured. This wasn't easy and if you don't believe me you can ask some of the senior boys. One of them told me they were drawing plans for houses. They had to figure the cost of their houses which took a lot of math, so what do you think a big school was like?

Sue: Golly, I bet it wasn't easy.

Jr.: Then, of course, I was always running up against the problem of time. The other kids kept asking me "What time is it? When does this period end? How many more minutes in this period?"

Jane: Yes a lot of math *is* connected with time. What are you doing in math now, Junior?

Jr.: Well, in math we are taking up percentage. Finding what per cent one number is of another, you know. They must think all kids are going to work in a bank, all the interest problems we have to do. And today we got an easy way to do them where you just move the decimal point two places to the left. It's called the 60 day 6% method. I guess it just works though for 60 days at 6%.

John: If you only had 30 days couldn't you just take half of it?

Jr.: I don't know if it would work or not. We didn't have any problems like that.

John: How about trying it to see. Let's find the interest on $700 at 6% for 60 days.

Jr.: Oh, that's easy. It's $7.00.

John: O. K. now let's make it 30 days, work it out the way you always have and see if the answer is $3.50.

(Junior works it out on a piece of paper, figuring out loud.)

Jr.: Let's see $700 times 6 over 100 times (pause) 30 days is 1/12 of a year so it would be times one-twelfth. Now cancel this and this and we have 7 over 2. That's 3 and one half dollars. It does, doesn't it! Now I guess I know *one* thing the other kids don't. Won't they be surprised when I start rattling off the answers to that kind of problems when we have them.

John: Now son, your friends may not appreciate your brains as much as you anticipate.

Jr.: In our discussion of where we saw math in our everyday life, one boy said he worked in the cafeteria and that Mrs. MacIlwee always has an awful time deciding how many potatoes to peel.

Sue: But where does all that stuff you've been writing down come in?

Jr.: Hold your horses, little sister, I'm coming to that. You know Jim and Sally Wilson that walk down the lane with us. I was telling Jim about our family project and he said "Just wait till you take Algebra! You can work the craziest sounding problems with that. Some that you couldn't make the least bit of sense out of now." I asked him to give me an example, so he asked me, "What are three consecutive numbers whose sum is equal to 3 less than 75?" He said you might be able to figure it out by trial and error if you had a long time. But he figured it out in just a little bit by letting x equal the first number and—and———Aw heck, I don't know how he got it but pretty soon he said the numbers were 23, 24, and 25. We added them up and they made 72 which is 3 less than 75. All right, Sue here is where the notes come in.

Sue: But how?

Jr.: Well, while walking down the lane he asked my why I didn't take notes on what we said tonight. He told me they might come in handy next year especially if I get Mrs. Fleming for Algebra. She makes them each turn in a project at the end of each term and one of the subjects they can take is *math in everyday life*. He said my notes might help him because that is his project now. It has to be in in a week and he's not near finished. I told him I'd take notes and he could see if he could use anything we said. I told him about our hay problem this morning, too, Dad, and he was almost overjoyed. He thought that would be a wonderful thing to represent the farmer with.

Sue: Where was I when you and Jim were talking about all that stuff?

Jr.: Aw, you and Sally were poking along picking flowers somewhere.

Sue: Yea, and Sally told me some stuff, too. Just wait till you hear that, Smartie. I'll get even with you. Sally said she is taking business arithmetic and they learn a lot of short cuts like multiplying two numbers, with something called fractions hooked on the end together. What's a fraction anyway?

John: That, dear, is like $\frac{1}{2}$ or $\frac{1}{4}$ of an apple. It's a part of a whole apple or a part of a whole anything.

Sue: Oh, I see. Then Sally said, too, that she works in the cafeteria. And one week they have to count how many lunches the kids buy and the next week they have to sell tickets. Their money has to come out right or they have to do it all over again and try to find a mistake.

(Sue stops for awhile and appears to be so engrossed in her thinking that she forgets to say anything.)

Jane: Did you have anything in arithmetic today that you want to tell us?

Sue: Oh, sure (becomes thoroughly alive again) we had some problems where we multiplied by two numbers like 78 multiplied by 24. I got mine all right and I don't think Bobby did.

Jr.: Did you learn anything more about telling time?

Sue: Yeah, and Miss Johnson gave us a 'rithmetic problem like Mother did this morning. She said, "If Johnny was sent to the store for a loaf of bread and he started out at 10 minutes after 4 and got back at 4:30; how long did it take him to go to the store and back?" I think I was the first one to get the answer. I just looked at the clock face we have hanging in our room and looked at ten after and then counted the minutes till I got to 4:30. It wasn't hard. I bet you can't guess what else we learned about time.

Jane: No, what?

Sue: Well, guess.

Jane: Oh, I guess Miss Johnson told you that people didn't always use clocks like ours but they used to use a sun dial.

Sue: No, she told us that when it's 9 o'clock here it's 6 o'clock in California. We had a map with the time belts on but you know I didn't see any buckles on those belts. (The family laughs at her.) Mother and I got the eggs this evening. I got the white ones and she got the brown ones. We got 188 in all and I got 60 of them. Mother, how many dozen more did you get than I did?

Jane: If junior will lend us a piece of paper we'll figure it out. (Junior tears off a sheet of paper and hands it to her.) First we better subtract 60 from 188. Let's see, 0 from 8 is 8 and 6 from 8 is 2 and bring down 1, that would be 128. That's how many I got. Now we'll divide 12 into 128. Twelve into 12 is 1, 12 into 8 is 0 and 8/12 or $\frac{2}{3}$. I got 10 $\frac{2}{3}$ dozen and you got 5 dozen. Subtract and I got 5$\frac{2}{3}$ dozen more than you did.

John: I saw an article in the paper the other day about how our

time belts in the United States were set up. I'll go see if I can find it.

Jr.: While you do that I'll go do my homework.

Jane: Will you help me clean up the dishes, Sue?

Sue: Sure.

SAVING SCIENCE RADIO PROGRAMS ON TAPE

HAROLD HAINFELD
Roosevelt School, Union City, N. J.

Once a radio program goes "off the air," it is usually difficult to borrow and impossible to keep a transcription of it for use in the class room. However there are some excellent science programs being broadcast that are worth saving for future school use.

Roosevelt School, located in the north-eastern part of New Jersey, is within range of two stations that broadcast programs designed for in-school listening. WNYE-FM, the New York City Board of Education station, and WBGO-FM, the Newark, N. J. school radio station, transmit science programs of value during school hours.

The use of our tape recorder has enabled us to start a library of useful science radio programs. If the program has enough merit for class room use, it can be saved for the future, when it is not available on radio, by recording it with our tape recorder.

Having a program recorded on tape offers solutions to other problems in radio utilization. Teachers are advised to preview films before use. By recording the program on tape, the teacher can pre-hear the program before using it in the classroom. If the program fails to meet his needs, the tape can be erased easily, and be available for another recording.

A program may be broadcast at one time of the year by the New ark or New York station, and yet fit our curriculum at another. A science broadcast may be in the morning and in departmental changing of classes, the teacher may not meet the students to hear the program until the afternoon. By recording broadcasts on tape, the best programs can be saved and used at the right time, as needed in the classroom.

During the past year we recorded programs from both WNYE and WBGO. "Pioneers In Science" was broadcast from New York, and "Science Lesson" from Newark. The former presented dramatically portions of the lives of scientists studied in the junior-high science and health class. "Science Lesson" was the audio portion of the television program designed to go with the recently adapted elementary science course of study in Newark.

We did not overlook making tape recordings from commercial radio stations in the evening to be brought to school. Most commercial stations, in addition to transmitting on AM wavelengths, also broadcast on static-free FM. We recorded from the FM band. It gives a higher frequency response and has almost no interference.

"Cavalcade of America," (WJZ in New York City—American Network) presented much useful material for the science program. With the program on tape, it was easy to edit and eliminate the commercial advertisements and announcements. The program, being sponsored by DuPont, often did contain valuable information about some new development. On a few occassions, these were placed at the end of the tape. This did not interrupt the continuity of the program, yet the information was available at the end of the tape for use, if desired. A 30 minute broadcast could thus be edited into a 23 minute tape recording, leaving plenty of time in the usual 45 minute class period for introducing the radio program to the students and to develop follow-up activities after the tape was heard. Having the evening program on tape guaranteed that it would be heard by all students. The students could be working, participating in athletics or other activities and miss the program at home when broadcast in the evening.

There is another possibility for the radio-tape recorder combination that we have used. WNYE presents a science quiz program where students from two junior or senior high schools in New York City try to answer questions on their science studies. Having the program on tape permits the teacher to let his students hear the question and stop the recorder before the answer is given. Thus the students in class can answer and discuss the question before the answer is given. This procedure would be impossible with just the radio.

In making tape recordings of radio programs, it is essential to have good equipment. This is not necessarily expensive. There are many fine AM-FM radio combinations available at low cost. A word of caution. Don't impair the quality of the recording by using a tape recorder with a frequency lower than the radio. A frequency response of from 50 to 10,000 cycles is needed for good quality recordings. These can be purchased by schools for two hundred dollars. Many recorders are two speed. It is possible to record at both $3\frac{1}{2}$ and 7 inches per second. The faster speed gives a much better quality when recording.

The storage of tapes is no problem. The reels are small and compact. The tape reels are approximately the size of 8 mm. motion picture film. Many photographic dealers have cans and containers for

the home movie-maker. This equipment can be used by schools to store their science radi programs.

Two recent developments can also aid the recording of science radio programs. The introduction of colored reels and tape can help in easily identifying the science tapes from others in the school library. Previously the reels were clear plastic and the tape an orange brown color. Now the teacher can store the science programs on blue or green reels or record on blue or green tape.

Excellent help has been given our project by the officials at WBGO and WNYE. Advanced program listings and contents of programs along with teacher's guides were forwarded to us. I feel sure that you can get the same cooperation from the educational broadcasters if you are within range of one of them. There are over 125 educational radio stations now broadcasting. Administrative approval of our project resulted in an increased budget for additional tapes this year.

Schools, school systems, county and state educational departments are developing film and audio-visual libraries. The possibilities of building libraries of valuable radio programs on tape should not be overlooked. Radio and recording equipment are not expensive items for the school budget. Building a library of taped radio programs that fit the curriculum is an excellent method of developing utilization techniques for these aids in teaching. The science teacher or department can lead in developing this type of project.

MAN HAS LIVED IN AMERICA FOR 24,000 YEARS

Man has lived in America twice as long as scientists have supposed.

Radiocarbon dating of fragments of charcoal found buried beneath an ash bed at Tule Springs, Nevada, and believed to be of "human origin" has pushed back the dawn of human life on American soil to more than 23,800 years ago. This date is included in a new compilation of radiocarbon dates issued by Prof. Willard F. Libby of the University of Chicago, now on leave to serve as a member of the Atomic Energy Commission.

Folsom Man who hunted now extinct mammoths in America's Southwest, and long believed to be the first American, has been dated by the radiocarbon method at about 10,000 years ago. Tepexpan Man, Mexico's first resident and contemporary of Folsom Man, has been dated at between 10,000 and 12,000 years ago.

But now it seems that man was living and perhaps cooking over a campfire in Nevada as much as 12,000 years before Folsom Man and Tepexpan Man were hunting their ancient elephant game.

The carbon sample was collected by Drs. Fenley Hunter and M. R. Harrington of Southwest Museum, Los Angeles.

Actually, the Nevada resident may have been much older. The antiquity of 23,800 years given by Dr. Libby for the Tule Springs sample is close to the limit of his radiocarbon dating method, so that he can only give the date as "older than 23,800." No one can now say how much older it may be.

A CHEMISTRY TEACHER REMEMBERS*

HE TRIED TO IMPROVE HIS EXAMINATIONS

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A teacher said his examination was pretty good except that it "flunked" all his good students and passed all his poor ones. His experience may have been a bit unusual even though such outcomes are a matter of record. He who has examined his tests critically has usually sensed a need for improvement.

Improvement implies a standard, a *best*, toward which one may progress. Perhaps, by conscious critical attention to each new test its results may be made more dependable and revealing. Some characteristics the master teachers have come to seek in their examinations are labelled: validity, reliabillty, objectivity and practicality.

As to validity; if one asks a student to count to one hundred his success in that requirement is not a valid index of his ability to read or to sing. Validity requires that the outcome of the test is a measure of the sort of achievement which the test set out to identify. That certainly ls a sensible requirement but it is truly surprising how many times the low validity of the test is not found by the teacher until the student has revealed it. The vocabulary of the question is frequently at fault. The direction to "State Boyle's Law" has frequently failed to get that law's statement because the student didn't know the historical association of the law with Boyle. Sometimes the question is "double-barrelled." A problem involving some principle of science may require the memory of a certain constant for its solution. The pupil fails to solve the problem because he didn't know the constant. Of course, if the intent was to test for his memory of the constant then it is a valid measure of the memory to call the answer wrong if he doesn't solve the problem. But why have the problem if the memory skill is all that is to be tested?

Another validity deficiency in examinations is due to an inadequate sampling of the subject-matter field. That is particularly difficult to attain if short-time essay examinations are used. To report upon a chemical analysis in terms of its significance there must certainly be a sample of genuine representative proportions. If a test of a student's understanding of acids is sought it is more adequate if questions on uses, preparation and properties are included than to ask only concerning preparation. It is this aspect of the examination which is improved when examinations of two or more hours in duration are arranged at the semester or quarter's end.

* This author presented a series of five short papers on tests in SCHOOL SCIENCE AND MATHEMATICS in the spring of 1947. See pages 114, 203, 322, 470 and 554 of those issues.

Reliability might be said to be an index of the test's dependability. Does this test, time after time, give the same rank order of the students of the class? There are at least two ways in which the reliability of an item may be checked; is the wording of the item so worded that it will always have the same meaning to the student even though he is asked to answer it at widely separate times? And is its answer sufficiently objective that the reader will not change his "model answer" between scoring sessions?

The question; "What is the meaning of the word 'normal'?" Every science teacher knows that "normal" means one thing in physics, another in chemistry and still something else in psychology. This is but another reminder that a test-maker needs to be at least sensitive to the relations of semantics to his task.

It is reported that the same examination paper in mathematics once drew grades all the way from 28 to 92% from a series of different teachers who scored it. This low reliability may have been due to careless phrasing of the requirements (which is not so likely in mathematics) or to reader's caprice. Certainly the answers were lacking in objectivity. Restated, an examination is rated highly objective if all readers who score it give it the same grade.

The word practical, when applied to tests, relates to: time required of the student in taking it, the time needed in scoring it; all considered in terms of its adequacy of sample and other considerations. In this connection the type of test selected for a given need as well as number of proctors needed, room space required and other secondary considerations are often involved.

Education is said to be a process of stimulating and guiding changes that are taking place in students. The teacher is desirous of knowing the extent of these changes. Attempts to gauge the magnitude of the changes are commonly determined by the aid of tests. However, before this exploration, it is needful to decide as to the kind of changes desired. Desirable changes, when determined, need to be described and catalogued. They should be described in such language that there will be little difficulty in recognizing them when they occur. In this search for desirable changes the teacher is probing for the objectives or aims of the course. If a teacher formulates and is committed to those aims he will exhibit much concern over their achievement. In consequence he will phrase those aims explicitly in terms of the changes he hopes to get. Their attainment and identification are his reward for his stimulation and guidance program of work.

Many times the statements of the aims are so general that they do not help to identify the evidence of the change in the direction of the aim. To illustrate: some teachers of chemistry wanted their students to have "a broad and genuine appreciation and understanding of the

world in which we live." Just what would a student do that would indicate he had such an "appreciation and understanding of the world"? Can the change from the student that does *not* to the one who *has* that "appreciation and understanding" be objectively described? Unless that can be done the attainment of that aim will have to be left to a very subjective evaluation. Thus one of the first lessons this teacher of chemistry had to learn was that his objectives had to be restated in terms of recognizable evidence for changes desired.

Much mouth service has been given to course aims with little follow-up in an effort to get evidence of their functioning. A group of some 200 chemistry teachers, 80% of them, gave approval to the aim: "Develop ability to draw conclusions from data." An inventory of their examinations showed less than 20% had made any effort to learn if their students had developed any skill in "drawing conclusions from data."

After having given much attention to the formulation of test items that were valid, reliable, objective and practical the chemistry teacher was loath to discard those items with but one use. It seemed an inexcusable waste of energy to thus abandon his best efforts. The result of that reluctance to part with his best lead to the gradual build-up of a magazine of "good" items for future use. He later, during a visit at the University of Minnisota, found a zoology teacher there who was following the same practice in his course.

In order to have some objective index of the "goodness" of his questions their characteristics, as revealed by student responses, were recorded upon them. Among these recordings were: difficulty, validity and, after many uses, its reliability. After a time such a reservoir became of much help in his job of test construction.

It should be noted that over the years he found that others, especially the examination committee of the American Chemical Society, could produce tests that were practical for his use even though he had to modify them to meet his local needs. But, perhaps, his greatest satisfaction in this phase of his teaching program came when he, like the old Domsie in the Scotch tale found, not a "lad of pairts" but a super-good test item to file away in a handy receptacle for use at some later time. However, he never did get away from speculation about and experimentation with tests for the intangible objectives.

Smooth shower heads, made of a nylon resin, have no chemical affinity for "lime" particles in hard water. This helps prevent the head from becoming plugged. The size of the jets can be adjusted and the water can be shut off at the head. The device screws on standard shower piping outlets.

ANALYSIS OF THE REQUIRED UNDERGRADUATE MATHEMATICS COURSES FOR ENGINEERING STUDENTS

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A. STATEMENT OF THE BASIC PROBLEM AND GENERAL OBJECTIVES

The primary difficulty in setting up a satisfactory sequence of courses for mathematics in an engineering college is the proper balancing of the utilitarian or applied aspects of the subject against its intrinsic value as a pure science. On the one hand we have the technical departments, such as physics, mechanical engineering and electrical engineering, whose main interest in mathematics revolves around problem solving techniques. The mathematics department, on the other hand, while admitting that the key position the subject enjoys in engineering education is due to its pragmatic values, cannot ignore the fact that mathematics has a coherent structure of its own which must not be neglected nor destroyed in setting up a curriculum for training engineers.

This basic problem is reflected in all the aspects of the teaching of mathematics in an engineering college from the daily lesson to the total curriculum. A well balanced program should be able to meet the demands of both viewpoints to such a degree that the student, while learning the techniques necessary for the applications, is also aware of the few really basic mathematical concepts which underlie the undergraduate course. The technical departments must in turn be convinced that the time spent in teaching these concepts is an investment which will make the student a better problem solver because of his surer knowledge of the logical structure of mathematics as an integrated body of principles rather than merely a set of enumerated theorems in the text. The teacher of mathematics for his part should have a broad background in physics or engineering so that he, while building the mathematical structure, can establish the connections between it and the technical subjects so necessary in engineering education.

A statement of the specific objectives of the required two year undergraduate mathematics curriculum in an engineering college should be given in terms of the ideal student as we would like to see him at the close of the sophomore year.

1. He should have a sound grasp of the following fundamental ideas of mathematics.
   a. The concept of a function.
   b. The concept of a limit.

c. The definition of a derivative.
d. The concept of a definite integral as the limit of a sum.
e. The meaning of the so-called "Fundamental Theorem of Calculus."

2. He should be thoroughly adept at the techniques of differentiation and integration.
3. He should be able to analyse a verbal applied problem into its component mathematical parts, to bring to bear his knowledge of principles and techniques to solve these parts and finally to integrate these mathematical solutions back into terms of the real environment which gave rise to it.
4. He should be aware of the areas of advanced mathematics lying beyond elementary calculus, such as differential equations, complex variable theory, probability, statistics, fourier analysis, vector analysis.

B. Term I

The major problems of Term I stem from the adjustment of the student to college and from the technical difficulty of teaching the elements of calculus in an introductory course.

The matter of adjustment to college is of course a highly complex problem in which the personality of the individual student interacts with an environment entirely new to him. Most students are able to adjust to this environment with degrees of difficulty which lie within tolerances acceptable to themselves and to the college community. A minority adjust only after some guidance and counseling, and a few individuals in spite of efforts both from themselves and from the college never successfully adjust to the college environment.

The teacher of freshmen must realize that the apparent failure of some of his students to learn mathematics may really be due to psychological and personality difficulties which are obscuring intrinsic abilities quite adequate for the mastery of the course. An ability to detect these students for reference to the available counseling authorities is greatly to be desired in the freshman teacher. But aside from this minority who need individual attention, the teacher must appreciate that most of his students, even the most capable, in the first few weeks of college life, are in a state of stress and apprehension. He should try to establish a classroom atmosphere of relaxed, objective, intellectual inquiry in which the student can as quickly as possible regain his normal sense of belonging and working effectively in a familiar group.

The primary objective of the subject matter content of Term I is the introduction of the differential and integral calculus. This may come as something of a shock to those who consider that a term or

even a year of analytic geometry, trigonometry, and algebra is necessary before engineering students can successfully learn calculus. However according to the recent report of the combined faculties of three leading preparatory schools and three leading universities,[1] one of the major sources of waste in our educational system is the unnecessary duplication in college of areas of knowledge already covered by high school courses. Surely mostly all students entering an engineering college will have had trigonometry and algebra, plus some experience with the basic ideas of analytic geometry. To repeat these courses in college as specific courses or even under pseudonyms such as "Introductory Mathematics" is an example of just this kind of waste. It is possible to construct a first term course in elementary calculus which, while including review work in these high school subjects as needed, has as its underlying purpose the breaking of new ground rather than the reworking of old.

An immediate service advantage of this early introduction to calculus is that it allows the Physics Department to present its subject from the vantage of having the derivative and integral as usable tools of analysis of physical principle. Thus it is possible to eliminate the educational waste involved in presenting college physics as a stale rehash of high school physics. However it is necessary that close liaison be established between the two departments, so that during that period of the first term while the Mathematics Department is leading up to the calculus, the Physics Department is teaching those branches of physics such as statics and elementary elasticity which do not require a knowledge of calculus for satisfactory college presentation.

There are three fundamental mathematical concepts to be introduced in Term I. They are:

1. The concept of a function.
2. The concept of a limit.
3. The concept of the definition of a derivative.

The first abstract idea of real importance to be encountered in Term I is that of the concept of a mathematical function, or rather the functional relation between two or more variables. The student is able to understand readily that any specific equation such as $y=3x^2+4x-5$ represents a functional relation between x and y. But he must be brought to understand that the general concept of functional relation goes far beyond such simple ideas and includes empirical graphs, tables of corresponding values, saw tooth waves, and strange mathematical functions which are defined verbally rather than algebraically. This broader picture gives him some perspective of the general power of mathematical thought.

[1] Harvard University. *General Education in School and College.*

The concept of a limit finds application almost everywhere in mathematics. It was one of the earliest problems which challenged the great genius of Archimedes. A really rigorous definition, replete with epsilons, would at this point tax the understanding of even the best freshmen. Yet the idea must be broached in connection with the definition of the derivative, if the latter is to have any understanding at all. The emphasis in Term I should be on specific numerical or functional examples of limits with only a limited attempt to present the general concept. But in succeeding terms, the matter should be brought up again and again, each time on a more rigorous level until the finishing sophomore has a sound understanding of this most basic concept.

The understanding of the definition of a derivative will proceed parallel to the undertanding of the idea of limit. However since the derivative is a specific case of the more general limit concept it is to be expected that more rapid progress can be made here toward the general concept from specific numerical and functional examples. However ability of the student to obtain derivatives from the use of the various differentiation formulas, should not be confused with understanding of the fundamental meaning of the derivative as a limit. Here too the teacher must return repeatedly to the really basic idea as distinguished from manipulative ability.

This is not to imply that the student should not be permitted to use the various formulas for finding derivatives until after he has mastered the abstract concepts of limit as applied to the derivative. There are innumerable life situations where the *how* of a procedure is known and used long before the more fundamental *why* is clearly understood. If a theoretical knowledge of the four cycle gasoline engine were required for the legal operation of a motor car our national traffic problem would be relieved instantly. What is unfortunate in the case of mathematics is that too many teachers are content with the *how* to the neglect of the *why*.

These ideas of Function, Limit, and Derivative are subtle and profound. It is not to be expected that the college freshmen will understand them on first exposure. But if it is a departmental policy to bring them up again and again throughout the two years of mathematics, with increasing attention to rigor and with repeated illustration by specific examples, then it is not unreasonable to expect a considerable absorption of their meaning by the end of the sophomore year.

A detailed statement of the Term I course is to be found at the end of the paper.

It will be noted from the detailed curriculum that a brief introduction to integral calculus is also made in Term I. This is mainly for

the benefit of the Physics Department so that they can use the technique of integration in the analysis of dynamics. The definite and indefinite integrals are introduced, but only from the viewpoint of the integral as the reverse process of differentiation. No attempt is made to approach the definite integral from its fundamental limit definition. This is of course regrettable, but the restricted penetration of the area of integration at this time plus the fact that the freshman is still struggling with the abstract concepts of function, limit, and derivative, seem to indicate the postponement of a more rigorous discussion of integration until Terms II and III. Here again the *how* precedes the *why*.

C. Terms II and III

Aside from some differentiation of transcendental functions in the beginning of Term II, the emphasis in these Terms II and III is on the powerful concepts and methods of the integral calculus. The fundamental ideas of mathematics to be discussed are:

1. The concept of the definite integral as the limit of a sum.
2. The "Fundamental Theorem of Calculus."

A most important task confronting the teacher of integral calculus is to disentangle in the student's mind (and of course his own) the usually confused concept of the indefinite integral, the definite integral, and the so-called Fundamental Theorem of Calculus.

The indefinite integral has not been listed here as one of the basic ideas of the course because it is simply an operation based on reversing the differentiation rules. Once differentiation is known, the indefinite integral follows at once. It has many applications in physics which is the reason it was introduced in Term I but it involves no more basic mathematical concepts than are already implicit in the derivative.

The definite integral on the other hand has its roots in mathematical antiquity. Archimedes attacked the problem of finding irregular areas by considering them as the sums of large numbers of small elements of area of regular shape. He applied the same method to volumes also, with considerable success. Other mathematicians and physicists before Newton's time used similar techniques throughout the history of science. Thus the concept of the definite integral antedates the concept of the derivative by some thousands of years. To drive home this fact the student should be shown the evaluation of some definite integrals such as

$$\int_a^b 1\,dx; \qquad \int_a^b x^2 dx; \qquad \int_a^b \sin x\,dx,$$

from the basic limit definition.[2]

The great source of confusion is that the familiar integral notation $\int dx$ is used both in the case of the definite integral, where it is justified, and also in the case of the indefinite integral where it is not. It is now too late to change this unfortunate misusage of symbolism, but its effect can be mitigated if the sharp historical and logical distinction can be made clear between the derivative and indefinite integral on one hand, and the definite integral on the other.

This distinction can be intensified by a lucid discussion of the Fundamental Theorem of Calculus, both historically and logically. First, it should be clearly established that Newton (or Leibnitz) did not invent calculus. We have already noted that Archimedes had been integrating in ancient times. Furthermore, Barrow, Newton's teacher, had already obtained the derivative geometrically. The great contribution of Newton-Leibnitz was to perceive that the definite integral, which was usually tremendously difficult to work out, each problem requiring some individual trick of technique, could be systematically and usually easily evaluated like an indefinite integral by reversing the differentiation process. That it is possible to do this should come as a great logical surprise, since the two problems of differentiation and evaluating the definite integral are not related *a priori* whatsoever. The surprise is of course not apparent to the student since the use of the integral sign for both the definite and indefinite integrals has conditioned him to the fact that the former can be evaluated by the same method as the latter. No pains should be spared to impress him with import of this great mathematical discovery which truly merits its name of The Fundamental Theorem of Calculus.

In these various applications of integration considered in Terms II and III the emphasis of learning should be on the adaptation to the problem at hand of the definition of the definite integral as the limit of a sum, rather than on the manipulative techniques of effecting the integration. This is not to say that training in the techniques should be neglected, but it surely should not be emphasized beyond its real importance. Most applied mathematicians when confronted with an integral whose solution is not elementary turn unashamedly to a good table of integrals. Yet we somehow believe that this privilege ought to be denied the student on the ground that since he is learning the subject he should be able to evaluate all integrals by analytic methods. This is really a form of academic hazing where the neophyte is made uncomfortable simply because he is a neophyte. If there is any intrinsic intellectual value in working out integrals, then everyone should do it.

[2] R. Courant, *Differential and Integral Calculus*, Volume I, pp. 82.

The real mathematical value in studying the applications of the integral calculus is to be found rather in the ubiquitous use of the definition of the definite integral as the limit of a sum in finding areas, volumes, first moments of area and mass, work, fluid pressure, etc. wherein the scope and power of this branch of mathematics is shown in its true richness.

D. Term IV

It can be seen from the detailed curriculum that the major topics of undergraduate differential and integral calculus can be covered in Terms I to III. The question as to what should constitute Term IV hinges on whether this term is to be a terminal course in mathematics or an introduction to higher applied mathematics.

Surely for those students who are going on into research and development engineering, or who may become physicists or even applied mathematicians, the choice is obviously for an introduction to higher applied mathematics. For those who are going on into industrial or sales engineering the question of how to terminate their formal instruction in mathematics would be extremely difficult to satisfactorily answer. Perhaps a broad course showing the historical development of applied mathematics from its crudest beginnings in antiquity to its elaborate utility in modern technical civilization would seem to be the answer. The objection to this would be that the student's lack of knowledge of mathematics beyond calculus would make it impossible to have a real understanding of the contemporary picture except in a most qualitative sense.

A more valid objection to such a procedure would stem from the fact that at the end of the sophomore year the student is not sure of his future area of specialization and hence cannot write off higher mathematics as unnecessary in his preparation. Furthermore even sales engineers occasionally have to read technical literature.

Finally, it might be argued that a proper terminal course in mathematics would itself consist of an introduction to the various fields of higher applied mathematics. The student who is not going further into the subject would then have at least a nodding acquaintance with these areas and be aware of the basic problems and techniques considered in them.

The methodology of teaching such a course is extremely critical as the teacher must steer a line between the one extreme of trying to cover as many topics as possible in the short time allotted to each subject field, and the other extreme of developing each topic so rigorously that too little ground is covered to gain the desired perspective and appreciation of the broad aspects of the field.

The choice of topics listed below is by no means exhaustive and

can be varied to suit the taste of the students and the instructor. If the student can be shown that the differential and integral calculus are the *San Salvador* rather than the *Ultima Thule* of mathematics, the objective of this term's work will have been achieved for both the terminal and the non-terminal student.

E. The Choice of a Text

Probably the best text written for any course is the one by the teacher himself based on several years' experience in that particular course. This is an ideal which is not, unfortunately, often realized in practice, and the usual procedure in choosing a text consists of a search of the current literature for a book closely approximating the material of the course and as congruent as possible with the philosophy of teaching of the instructor.

There are several texts available for the course of the first three terms as outlined below. The introduction of the calculus at the earliest possible time after a brief look at analytic geometry is attracting the attention of competent authors more frequently than in the past. Unfortunately, there are not many books available for the Term IV type of course but some exist which could be used with some careful selection of topics and supplemental notes by the teacher.

However, the philosophy of the author is equally important as his choice of topics. On the one hand we have the mathematician of extreme rigor whose primary concern is with the precision and elegance of his statements. At the other extreme is the author who in his sincere desire to make everything so lucid and understandable oversimplifies subtle and complex ideas to the point of loss of the necessary exactness which is germane to mathematics as a science.

A master teacher however can teach a good course using either type of text. In the first case he must lead the students up to the level of the text by classroom discussions which clarify the aphoristic language of the text. In the other situation the teacher must himself furnish the rigor necesssary for the adequate mathematical treatment which the text does not attain.

In any case it should be clearly recognized that the teacher rather than the text sets the tone of the course. Some of the best teaching can be done without a text at all, thus avoiding the inevitable conflict between the author and the teacher over the general procedure of presentation of the course. However, surely for freshmen and probably for sophomores, the presence of a well written text acts as a kind of reserve to which the student can turn to clarify questions and obscurities remaining from the class meeting. Furthermore, one of the legitimate aims of college education is the training of the student to

read scientific writing without prior discussion in class of the material to be covered. Here too a good text can be used to advantage.

MATHEMATICS CURRICULUM

TERM I

I. Review of Algebra
   1. Fundamental Operations
   2. Factoring
   3. Linear Equations
   4. Laws of Exponents
   5. Quadratic Equations
   6. Binomial Theorem

II. Rectangular Coordinates
   1. Location of points, directed lines
   2. Distance between points
   3. Angles, Slopes
   4. Straight Line
   5. Plotting Curves
   6. Derivation of Equations

III. Differential Calculus
   1. Variables and Functions
   2. Tangent to a Curve, Instantaneous Velocity
   3. Limit of a Function
   4. Derivative
   5. Derivative of Algebraic Functions, Chain Rule
   6. Applications
   7. Differentials
   8. Indefinite Integral

IV. Definite Integral
   1. Areas under Curves
   2. Volumes
   3. Work, Mean Value

V. Extreme Values
   1. Increasing and Decreasing Functions
   2. Extreme Values

VI. Conic Sections
   1. Circle
   2. Parabola
   3. Ellipse
   4. Hyperbola

VII. Curve Tracing
   1. Intercepts, Symmetry, and Extent
   2. Aymptotes, Points of Inflection
   3. Curve Tracing

TERM II

I. Differentiation of Transcendental Functions
   1. e^x and $\log x$
   2. Trigonometric Functions
   3. Inverse Trigonometric Functions

II. Theorem of the Mean and Indeterminate Forms

III. Review of Definite Integral
   1. Definition as Limit of a Sum
   2. Fundamental Theorem

IV. Techniques of Integration
   1. Standard Forms
   2. Trigonometric Functions

3. Integration by Parts
4. Partial Fractions
5. Use of Tables

V. Further Applications of the Definite Integral
1. Review with Emphasis on Definition and Fundamental Theorem
2. Arcs, Areas, Volumes
3. Centroid, Second Moments, Work
4. Numerical Methods

VI. Polar Coordinates
1. Analytic Geometry of Polar Coordinates
2. Calculus of Polar Coordinates

VII. Parametric Equations
1. Parametric Equations
2. Curvilinear Motion
3. Integration and Areas
4. Locus Problems

VIII. Curvature
1. Definitions
2. Circle and Center of Curvature
3. Evolutes, Involutes

TERM III

I. Definite Integral
1. Review of Definition and Fundamental Theorem
2. Centroids, Second Moments
3. Polar Coordinates
4. Numerical Methods

II. Series
1. Convergence and Divergence
2. Tests
3. Power Series
4. Taylor's Series

III. Linear Equations
1. Determinants, Minors, Cofactors
2. Cramer's rule

IV. Solid Analytic Geometry
1. Points, Distances, Directions
2. Planes, Lines
3. Surfaces

V. Partial Differentiation
1. Partial Derivatives
2. Total Differential
3. Chain Rule
4. Envelopes, Tangent Lines
5. Maxima and Minima

VI. Multiple Integrals
1. Double Integrals
2. Polar Coordinates
3. Surface Areas
4. Triple Integration
5. Cylindrical and Spherical Coordinates

TERM IV

I. Complex Numbers
1. Definition
2. Powers and Roots
3. Exponential Form
4. Uses in Integration

II. Differential Equations
  1. First Order
  2. Second Order Linear with Constant Coefficients
  3. Vibrating Systems
  4. General Linear with Constant Coefficients
III. Fourier Series
  1. Derivation of Coefficients
  2. Use in Vibration Problems
IV. Introduction to Statistics
  1. Probability
  2. Data Representation
  3. Correlation
  4. Distributions
  5. Statistical Inference

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A REVIEW OF CANCER: A RESEARCH STORY

Cancer: A Research Story, 16 mm., color, sound, 25 minutes. A free loan film available from American Cancer Society, Inc., 47 Beaver Street, New York 4, N. Y. or from American Cancer Society, Illinois Division, Inc., 139 N. Clark, Chicago, Ill.

Commentary by Dr. Charles S. Cameron, medical director of American Cancer Society. A very recent release which shows how the cancer problem is being attacked by various phases of research. Heredity, viruses, hormones, carcinogenic chemicals, and selective destruction of cancer cells, are being investigated as causative agents at research centers. The use of experimental animals is the very basis of the various attacks on the entire cancer problem. The spectator sees famous specialists in research fields in their own laboratories. It presupposes a background of general information on cancer and should not be shown below the high school level.

James M. Sanders

AN ANALYSIS OF MATHEMATICS COURSES IN FOUR YEAR COLLEGES

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Are mathematics courses in various colleges in the United States equivalent in such matters as content, title, hours of credit, prerequisites, and level at which offered? Is there any uniformity in the requirements for a mathematics major? There seems to be little, if anything, in the literature which would pertain to these questions. The problems are particularly pertinent in determining eligibility of transfer students wishing to take work in mathematics. It is the purpose of this study to compare course offerings in mathematics and requirements for a mathematics major in four year colleges in the United States.

If a mathematics department decides to analyze its course offerings, some questions that could arise are: Is the proposed course comparable to similar courses in other colleges—i.e., does it cover the same content; is it at the same student level; are the prerequisites similar; are the credit hours essentially the same? Is this course commonly offered at four year colleges? Is the proposed course title descriptive of the content; will it have meaning if it appears on a transcript? Is the combination of subject material one which is usually found in four year colleges?

The labor involved in studying the mathematics curricula for every four-year college would be prohibitive, hence some sampling method seems advisable. In order to determine the colleges and universities from which the sample would be taken the *Education Directory*[1] published by the U. S. Office of Education was used. This lists institutions classified by level of training and type of program. Institutions with a program below the bachelor's degree and certain specialized institutions (for example, theological schools, schools of chiropody) were excluded.

Within each of the remaining categories as listed in the *Education Directory*, a ten per cent sample was taken by making use of *Tables of Random Sampling Numbers*[2] by Kendall and Smith. This gave 113 schools which seemed a representative sample in spite of the fact that some of the schools are not widely known. However, casual inspection of the entire list gives the impression that the majority of the schools are known locally rather than nationally.

* Based on a thesis presented for the master's degree at the University of Wichita, Wichita, Kansas.
[1] *Education Directory, Part III, Higher Education, 1952–53.*
[2] M. G. Kendall and B. Babington Smith, *Tables of Random Sampling Numbers*, 1939.

The information used in this study was found in catalogues published by the schools in question. If for some reason a catalogue was not available, a letter was sent to the head of the mathematics department asking for the required information. Of the 113 schools in the sample, information was obtained from 98 of the schools which represents 87% of the schools in the survey.

Since some schools are on the quarter system rather than the semester basis, to make the results comparable all quarter hours were converted to semester hours using the relation that one quarter hour equals two-thirds of a semester hour.

The number of courses offered at a school ranges from zero to ninety-one. The mean number of courses offered is sixteen. As might be expected all the schools offering more than thirty courses (less than 10% of the schools) also offer graduate work.

Requirements for a Mathematics Major

There appears to be no common practice in the requirements for a mathematics major. Most schools specify that a year of calculus must be taken to satisfy the requirement. Assuming that this implies from fourteen to twenty hours have been taken in mathematics, the most common requirement for a mathematics major seems to be approximately twenty-four semester hours of mathematics. Frequently additional specified courses are required, but there is no general agreement as to what these additional courses should be. One finds such requirements as advanced calculus, differential equations, modern geometry, theory of equations, history of mathematics, statistics, higher algebra, and general physics. The lowest requirement for a mathematics major is twelve semester hours. The highest requirement is forty-eight semester hours. The only two colleges requiring courses in fields other than mathematics are women's colleges. (In both cases the requirement is general physics.)

Plane Geometry

Only six schools offer plane geometry. One school offers two courses in the subject, thus seven courses can be analyzed. Topics mentioned in the catalogue descriptions of a plane geometry course include congruent triangles, parallels, perpendiculars, construction of triangles, areas, chords and tangents to circles, loci, concurrence, similar triangles and polygons, and proportionals.

Except for one course the plane geometry courses are very similar. Six of the courses are offered on the freshman level, with one year of high school algebra as the prerequisite. The mean number of hours of credit is three hours. Only one school offering the course fails to give college credit.

Solid Geometry

Eighteen courses are entitled Solid Geometry. All of the courses are offered in the freshman year and all of the courses have a prerequisite of one year of high school geometry. All the schools offering the course allow college credit. Seven schools offer the course for two hours of credit; eleven schools for three hours. Topics mentioned include lines and planes in space; three dimensional figures such as polyhedrons, cylinders, prisms, cones, pyramids, and spheres; dihedral and polyhedral angles. Three schools include spherical geometry in the content; two schools include spherical trigonometry in the content.

General, Basic, or Fundamental Mathematics

There is very little uniformity in content and hours of credit in a course entitled General, Basic, or Fundamental Mathematics. A course coming under this heading may include mathematics of finance, intermediate algebra, college algebra, trigonometry, or even beginning calculus. In some cases the entire content of the course may be mathematics of finance, or again intermediate algebra, or possibly college algebra. It seems that a course title by no means is indicative of the course content. A course with any of the above titles may carry from two to ten hours of credit. In no case is there a stated prerequisite and in every case the course is offered on the freshman level.

First Year Algebra

In the twelve schools offering a first course in algebra eight different titles are used: Sub-Freshman Algebra, Elementary Algebra, Remedial Mathematics, Mathematics, Algebra, High School Review, Introduction to Algebra, and Principles of Mathematics. The course is offered only on the freshman level. Besides including "the content of first year alegbra" (which no catalogue defines) eight schools also include the essentials of arithmetic and geometry. Even though the course is essentially a high school course five of the schools offering the course grant college credit; three to four semester hours.

Intermediate Algebra

Twenty-five courses can be classified as intermediate algebra. Titles used are Intermediate Algebra, Algebra, Introductory Algebra, Elementary Algebra, Freshman Mathematics, Advanced Algebra, and Survey Course in Mathematics. All the schools offer credit in the course—nineteen colleges offer it for three hours of credit; three for two hours; two for four hours; and one for six hours.

Thirteen schools definitely state there is a prerequisite of one year of high school algebra. (Of these one also includes one year of high

school geometry.) All the courses are offered on the freshman level.

If the twenty-five courses analyzed two are described as "first course in algebra" and eight as "third semester algebra" without further description of content. In the remaining course descriptions, topics mentioned are quadratic equations, linear equations, graphs, factoring, exponents, functions, fractions, radicals, systems of linear equations, progressions, logarithms, binomial expansion.

It appears that an intermediate algebra course normally includes quadratic equations, linear equations, graphs, factoring and probably exponents. If one may judge by catalogue descriptions the inclusion of logarithms and the binomial expansion seems rare.

Specialized Beginning Mathematics Courses

There are seven mathematics courses for students in specialized fields—agricultural mathematics, mathematics for nurses, mathematics for architects, mathematics for medical technologists. All the courses are on the freshman level. Topics covered vary from arithmetic to beginning calculus (in some cases statistics). The hours of credit range from two to six.

College Algebra

Sixty-eight courses can be classified in the category of college algebra. The most common title is College Algebra; other titles are Algebra, Advanced Algebra, and First Year College Mathematics. Sixty-three of the courses are offered on the freshman level; the remainder on the sophomore level. Only thirty-four courses state definite prerequisites. Twenty-six of the courses require intermediate algebra or at least one and one-half units of high school algebra. Seven courses have as prerequisite one unit of high school algebra. Plane trigonometry is the prerequisite of one course.

Twenty courses have no course description or state "the usual course in College Algebra." The courses which list subject matter, mention twenty-three different topics: determinants—thirty-seven times; theory of equations—thirty-six times; progressions—thirty-five times; binomial theorem—thirty-five times; quadratic equations—thirty-four times; permutations, combinations, probability—twenty-eight times; logarithms—twenty-four times; variation—twenty-four times; linear equations—nineteen times; complex numbers—seventeen times; graphs—sixteen times; mathematical induction—fifteen times; ratio and proportion—fourteen times; exponents, radicals, fractions—eleven times; inequalities—eleven times; systems of equations—eleven times; partial fractions—ten times; series—five times; compound interest and annuities—five times; statistics—once.

One course is offered for two hours of credit. Forty-six schools offer the course for three hours of credit; seven schools for five hours; and five schools for six hours.

Thus a "typical" college algebra course would seem to carry three hours of credit, be offered on the freshman level, either have intermediate algebra or one and one-half units of high school algebra as prerequisite, and include such topics as determinants, theory of equations, progressions, binomial theorem, quadratic equations, permutations and combinations, probability, logarithms, and variation.

Trigonometry

Sixty-one schools offer a course in plane trigonometry; fifty-three for three hours of credit; seven for two hours; one for four hours. Fifty-seven of the courses are offered on the freshman level; four on the sophomore level. Thirty-nine schools state definite prerequisites: sixteen require intermediate algebra; nineteen require college algebra; four require college algebra or concurrent enrollment.

Fourteen schools give no catalogue description of plane trigonometry. Of the course descriptions obtained, forty-two include solution of triangles. Other topics mentioned are: Trigonometric functions—thirty-seven times; logarithms—thirty-one times; identities—twenty-four times; trigonometric equations—twenty-one times; inverse functions—fourteen times; graphs—eleven times; radian measure—ten times; addition, half angle and reduction formulas—eight times; complex numbers—five times.

If one can identify a "typical" course in plane trigonometry it is for three hours of credit, has at least intermediate algebra as prerequisite and is open to freshmen. Topics covered in this typical course are trigonometric functions, solution of triangles, logarithms, trigonometric equations, identities, inverse functions, radian measure, and graphs. A single course in plane and spherical trigonometry is a distinct rarity.

Advanced and Spherical Trigonometry

Eleven schools offer a course entitled Spherical Trigonometry or Advanced and Spherical Trigonometry. The semester hours of credit vary from one to four. The course is most commonly offered on the sophomore level (seven schools). Ten schools require plane trigonometry as prerequisite; two require college algebra in addition to plane trigonometry; and one requires analytic geometry in addition to plane trigonometry.

The ten schools giving course descriptions all include solution of spherical triangles. Other topics mentioned are navigation, astron-

omy, DeMoivre's theorem, series, trigonometric equations, and hyperbolic functions.

Surveying, Navigation, Astronomy

Eleven schools offer a course in surveying; three each offer a course in navigation and astronomy. The three schools offering astronomy also offer work beyond the bachelor's or first degree. Only one of the sixteen courses could not be counted upon a mathematics major. Most of the courses are offered on the sophomore level with the hours of credit varying from two to six.

Analytic Geometry

Fifty-nine schools offer a course titled either Analytic Geometry or Plane Analytic Geometry. One school offers the course for two hours of credit; thirty-four for three hours; twelve for four hours; nine for five hours; and three for six hours. Thirty schools open the course to freshmen; twenty-six schools to sophomores; and three schools to juniors. (These three are teacher preparatory schools, not offering a degree beyond the bachelor's degree.) Sixty-one per cent of the schools in the sample offering degrees beyond the bachelor's degree open the course to freshmen.

Fifty of the fifty-nine courses state prerequisites. The most common prerequisite (fifty-four schools) is plane trigonometry and college algebra. Only one school specifically states a minimum quality of work in a prerequisite course.

Forty-three catalogues give course descriptions with all mentioning conic sections. Other topics are coordinate systems, rectangular and polar—thirty-six times; straight line—thirty-five times; transformation of coordinates, translation and rotation of axes—twenty-five times; parametric equations—twelve times; higher degree equations—ten times; tangents and normals—nine times; curve tracing six times.

Nineteen schools include in the first course a study of solid analytic geometry. Twenty-eight schools offer a distinct course called Solid Analytic Geometry. All but one of the twenty-eight are for three hours of credit. Two courses in solid analytic geometry are open to freshmen; one to sophomores; twenty-five to juniors or seniors.

Twenty schools state prerequisites for solid analytic geometry. Nine require the first course in analytic geometry; six differential calculus; two integral calculus; one the first course in analytic geometry and advanced trigonometry; one the first course in analytic geometry and consent of department; and one theory of equations.

Twenty-two descriptions of solid analytic geometry are given.

Twenty schools include plane and straight line. Other topics are surfaces, space coordinate systems, transformations, second degree equations, and less frequently: matrices, determinants, invariants, volumes, parametric equations, cross ratios, and duality.

There are perhaps two "typical" courses in analytic geometry. The most common course is for three thours of credit and includes a study of the straight line, conic sections, transformation of coordinates by translation and rotation, rectangular and polar coordinates. Only six of the thirty-five schools offering the course for three hours of credit or less indicate that solid analytic geometry is included in the first course.

There is also an appreciable group of schools which offer analytic geometry for four, five or six hours of credit. In general such courses include the content of the three hour course and in addition the topics of tangents and normals and of solid analytic geometry are studied. Only twenty percent of the courses with three or less hours of credit include solid analytic geometry, whereas fifty-eight per cent of the courses with four or more hours of credit include solid analytic geometry.

Only six of the twenty-eight distinct solid analytic geometry courses follow a first course in analytic geometry offered for more than four hours of credit. In general a solid analytic geometry course is offered only as a distinct course when the first course carries three or less hours of credit.

Calculus

There seem to be two distinct plans for presenting beginning calculus, either in two courses or three courses. In all there are eighty course sequences (some of two, others of three courses) that seem to cover beginning calculus, both differential and integral. Three of these sequences total four hours of credit; one five hours; twenty-seven six hours; one seven hours; thirty-one eight hours; nine nine hours; and eight ten hours. Sixty-five schools offer beginning calculus in two courses, fifteen schools in three courses.

First Course of Two Courses in Calculus: This course is essentially differential calculus. Three courses are offered for two hours of credit; thirty for three hours; twenty-five for four hours; seven for five hours. Seven courses are open to freshmen; thirty-six to sophomores; twenty-two to juniors. Of the forty-nine courses stating prerequisites forty-seven have a prerequisite of analytic geometry.

Fifty catalogues mention topics one studies in the first course in calculus. Topics included are differentiation and derivatives; functions, limits, continuity; maxima and minima, point of inflection; indeterminate forms; curvature, curve tracing. Applications to geome-

try and physics are heavily stressed. Thirty per cent of the courses include an introduction to methods of integration.

Second Course of Two Courses in Calculus: This course is primarily integral calculus. Three courses are for two hours of credit; thirty-one for three hours; twenty-five for four hours; six for five hours. Four courses are open to freshmen; thirty-four to sophomores; twenty-six to juniors; one to seniors. Fifty-eight courses require the first course in calculus as prerequisite.

All the second courses in calculus include methods of integration. As in the first course applications are stressed. Topics discussed are definite integrals, fundamental theorem of integral calculus, multiple integrals, indeterminate forms, differential equations and partial differentiation.

First Course of Three Courses in Calculus: When the sequence in beginning calculus involves three courses, the first course is primarily differential calculus. All the courses are essentially for three hours of credit. Three of the fifteen courses in this group are open to freshmen; seven to sophomores; five to juniors. All the courses require analytic geometry as a prerequisite.

Topics discussed are differentiation and differentials, limits, indeterminate forms, maxima and minima, curve tracing, tangents and normals, and curvature. Thirteen of the fifteen courses state that applications are stressed. Only four courses introduce integration in this first of three calculus courses.

Second Course of Three Courses in Calculus: This course is primarily integral calculus. Again all the courses are essentially for three hours of credit. Nine of the courses are open to freshmen; six to juniors. All the courses require the first course in calculus as prerequisite.

Third Course of Three Courses in Calculus: This course seems to be a continuation of integral calculus with applications heavily stressed. The course is essentially for three hours of credit. Seven courses each are on the sophomore level and the junior level. One is on the senior level. All require the second course in calculus as prerequisite.

Applications that are stressed in this course are centroids, moments of inertia, pressure, work, areas, volumes, rates. Other topics discussed are series, partial differentiation, multiple integrals and differential equations.

Combined Courses

Many schools instead of offering separate courses in various subjects offer what might be called a unified or combined course. There are varied titles assigned to such courses such as Unified Mathematics, Mathematical Analysis, First Year Mathematics and sub-

ject titles. Table I lists these courses with certain pertinent information.

Advanced Calculus

Of the courses entitled Advanced Calculus one carries two hours of credit; twenty-seven carry three hours; five carry four hours; three carry five hours; twenty carry six hours; and three carry eight hours. Twenty-five courses are open to juniors; forty-one to seniors; three to graduate students. Of the fifty courses with stated prerequisites, all require integral calculus.

Table I. Combined Courses

Subjects Included	No. of Courses	Most Common Hours of Credit	Year Offered	Prerequisites
College algebra, plane trigonometry	16	3	freshman	intermediate algebra
College algebra, plane trigonometry, analytic geometry	14	7	freshman	intermediate algebra
College algebra, analytic geometry	2	6	freshman	2 years high school algebra
College algebra, plane trigonometry, analytic geometry, differential calculus	7	6	freshman	2 years high school algebra
College algebra, plane trigonometry, analytic geometry, differential and integral calculus	2	10	sophomore	none stated
Plane trigonometry, analytic geometry	4	5	freshman	college algebra
Plane trigonometry, analytic geometry, differential calculus	1	6	freshman	college algebra
Analytic geometry, differential calculus	6	5	freshman	college algebra, plane trigonometry
Analytic geometry, differential calculus, integral calculus	5	10	sophomore	none stated
Advanced analytic geometry, integral calculus	3	6	freshman	analytic geometry, differential calculus

Sixty-five per cent of those schools offering graduate work offer a course in advanced calculus. Forty-one per cent of the schools offering only the bachelor's degree offer advanced calculus. Most advanced calculus courses contain multiple integrals, partial differentiation, series, line and surface integrals, improper and definite integrals, applications, Fourier series, continuity and limits. Less frequently an advanced calculus course covers such topics as complex variables, differential equations, vectors, gamma and beta functions, Bessel functions, hyperbolic functions, elliptic integrals and calculus of variations.

Differential Equations

Eighty courses entitled Differential Equations are found in this survey. The most common number of hours of credit is three (sixty courses). Two courses are apparently open to freshmen; forty-one courses to juniors; thirty-two to seniors; five to graduate students. Of the sixty-two courses with stated prerequisites, fifty-six courses have a prerequisite of integral calculus. Six courses require advanced calculus.

Topics covered in a differential equations course are first and second order differential equations, partial differential equations, and solution by series. Applications to geometry, physics and mechanics are heavily stressed. Topics less frequently covered (almost exclusively in courses open only to graduate students) are Fourier series, Bessel and Legendre functions, Bernoulli equations, vectors and Heaviside's calculus.

Theory of Equations

Sixty-two courses come under the heading of theory of equations. This course is essentially a three hour course. Only four courses grant credit other than three hours. The course is almost exclusively open only to juniors and seniors.

The most common prerequisite is differential calculus. Of the forty-five courses with stated prerequisites eighteen give differential calculus as prerequisite; twelve give integral calculus; four each give college algebra and analytic geometry; three give higher algebra; two give differential calculus or concurrent enrollment.

Topics most frequently taken up are determinants; complex numbers; cubic and quartic equations; theorems on roots of equations; higher degree equations; approximate roots, Newton's and Horner's methods; systems of linear equations; symmetric functions; ruler and compass constructions; and matrices.

Advanced and Higher Algebra

Thirteen courses are entitled Higher Algebra and nine courses

Advanced Algebra. Fifteen courses are for three or less hours of credit; seven are for four or more hours. All the courses are upper division or graduate courses with four open only to graduate students. Fifteen courses have stated prerequisites with integral calculus the most common.

The content of a course in advanced or higher algebra varies. The topics most frequently mentioned are: determinants, matrices, groups, systems of equations, partial fractions and series. Topics mentioned but once or twice are inequalities, mathematical induction, probability, summation, undetermined coefficients, invariants, fields, congruences, quadratic forms, vector spaces, quaternions, number systems, exponents, permutations and combinations.

Modern Algebra

Twenty-eight schools offer a course in Modern Algebra. Seventy-five percent of these are schools offering work beyond the bachelor's degree. Sixteen courses are for three hours of credit; the remaining range from five to nine hours. Fifteen courses are open to juniors and seniors; thirteen are graduate courses. Of the nineteen courses with stated prerequisites sixteen require integral calculus.

A course in modern algebra generally includes matrices, determinants, groups, rings, fields, ideals, number theory, congruences, invariants, and vector spaces.

Theory of Numbers

Twenty-nine courses are entitled Theory of Numbers or Number Theory. All but five of these courses are offered for three hours of credit. Fifty-two percent of the courses are graduate courses; the remainder are open to juniors and seniors. In general those courses open to graduate students have a prerequisite of integral calculus; those open to undergraduates require differential calculus. Topics usually covered in this course are properties of whole numbers, divisibility, congruences, quadratic residues, reciprocity, Diophantine equations, quadratic forms, continued fractions and prime numbers.

Modern or College Geometry

Fifty-five courses are entitled either College Geometry, Modern Geometry or Higher Geometry. Forty-seven courses are for three hours of credit. All but four courses are upper division or graduate courses. Thirty-four schools state definite prerequisites with analytic geometry as the most common prerequisite.

Topics covered include similarity, the triangle and its notable points, circles, the nine point circle, harmonic division, poles and

polars, coaxial circles, inversion, homothetic figures, constructions, and cross ratio. Nine courses indicate that a course of this nature would be helpful to the teacher of mathematics.

Projective Geometry

Twenty-eight schools offer a course in Projective Geometry. Twenty-three of the courses are for three hours of credit. All the courses are upper division courses with seven open exclusively to graduate students. Of the twenty-one courses stating prerequisites, integral calculus is the most common. Seven schools indicate the course is synthetic projective geometry; four schools indicate the course is analytic projective geometry. Topics studied are projective properties of curves and surfaces, conics, cross ratio, duality, theorems of Desargue and Pascal.

History of Mathematics

Twenty-two schools offer a course in the history of mathematics. Four courses are for two hours of credit; eighteen are for three hours of credit. One course is a graduate course; the remainder are junior and senior courses. One course requires differential calculus as prerequisite; seven require integral calculus; one requires six hours of mathematics; two require consent of the department. Two schools indicate that the history is taught through analytic geometry; four indicate that it is taught through the calculus. Most courses are in agreement that the historical periods of mathematics are studied with an attempt to relate the various fields of mathematics.

Vector Analysis

Twenty courses are entitled Vector Analysis. Sixteen courses are for three hours of credit. One course is a graduate course; the remaining are junior and senior courses. Of the sixteen courses stating prerequisites, fifteen courses require integral calculus. Three courses require advanced calculus and one course requires theory of equations in addition to integral calculus. Topics discussed are gradient, divergence, curl, differentiation and integration of vectors; integral transformations, Stokes' and Green's theorems, with emphasis on applications. Two schools include tensor analysis in a course with this title.

Complex Variables

The seventeen schools offering a course in Complex Variables all offer work beyond the bachelor's degree. Nine courses are for three hours of credit; two are for four hours; six are for six hours. Five

courses are open to seniors; the remaining are graduate courses. The most common prerequisite is advanced calculus. Topics discussed are conformal mapping, series, transformations and analytic functions as applied to the complex domain.

Mechanics

Fifteen schools offer a course in Mechanics. Nine courses are for three hours of credit; one is for four hours; five are for six hours. Two courses are open to sophomores; eleven to juniors and seniors; two to graduate students. Of the eleven courses with stated prerequisites, one requires college algebra; one analytic geometry; one differential calculus; six integral calculus; two differential equations. Only three courses require general physics. Topics discussed are motion of a particle, dynamics, statics, and kinematics.

Mathematics of Finance and Business Mathematics

Thirty-three courses are entitled either Mathematics of Finance or Mathematics of Investment. One course is offered for two hours of credit; twenty-eight are for three hours; four are for four hours. Sixteen courses are open to freshmen; ten to sophomores; seven to juniors. Of the twenty-three courses with stated prerequisites, two require intermediate algebra; twenty require college algebra; one requires analytic geometry. Practically all the courses include the topics of interest, annuities, bonds, insurance, depreciation, amortization, sinking funds and discounts.

There are nineteen courses entitled Business Mathematics. In general this course is for three hours of credit and open to freshmen. Seven courses have stated prerequisites, six requiring intermediate algebra and one college algebra. In analyzing the content it is found that three courses cover the same material as is usually found in a course called Mathematics of Finance. Four courses cover the material covered in college algebra. Four other courses cover material found in both mathematics of finance and college algebra. The remaining eight courses fail to indicate their content. It appears that a course entitled Business Mathematics may vary markedly in its content.

Statistics

There are fifty-nine first courses in statistics offered in schools in this survey. Forty-three are for three hours of credit; five are for two hours; three are for four hours; eight are for six hours. Eighteen courses are lower division; thirty-nine are upper division; two are graduate courses. Of the forty courses stating prerequisites, two require intermediate algebra; eighteen college algebra; four analytic

geometry; fourteen integral calculus; one advanced calculus; one consent of department.

Since there seem to be two different prerequisites to a first course in statistics (college algebra or calculus) analysis of the content of statistics was made in two parts. Those courses with intermediate algebra, college algebra, or analytic geometry as prerequisite include the topics of frequency distributions, graphs, measures of central tendency, measures of dispersion, measures of variability, normal curve, sampling theory and simple correlation.

Those courses with a prerequisite of at least calculus include in the content, along with the topics already mentioned, moments, method of least squares, curve fitting, partial and multiple correlation and tests of hypotheses.

There are twenty-seven second courses in statistics which have as a prerequisite the first course. Four are for two hours of credit; fourteen for three hours; four for six hours. Two are lower division; sixteen are upper division; four are graduate courses. In addition to the topics mentioned in the first course, other topics considered are sampling theory, other types of correlation, tests of statistical hypotheses, and probability.

Teaching of Mathematics

Only those schools offering a teacher preparatory course offer courses in the teaching of mathematics. However, not all schools offering a teacher preparatory course offer such courses. Twenty-two schools offer a course in the teaching of elementary mathematics. Eleven courses are lower division; seven are upper division; four are graduate courses. In general there is no prerequisite to this course. The subject matter consists of the methods of teaching arithmetic.

Thirty-two courses are concerned with the teaching of mathematics on the secondary level. This is either a two or three hour course with a prerequisite of approximately twelve hours of mathematics.

Seven courses fail to indicate whether the course is for elementary or secondary school teachers. Eight schools offer a separate course for the elementary teacher and the secondary school teacher.

Seminars and Reading Courses

Eighteen schools offer a mathematics seminar; thirty-four a course in independent reading; five thesis. Most of the courses are either on the senior or graduate level. The hours of credit vary from one to ten hours. In very few cases are there stated prerequisites.

Miscellaneous Courses

In addition to the courses already stated there are many miscel-

laneous courses offered in the schools studied. These courses are listed in Table II.

TABLE II. MISCELLANEOUS COURSES

Course Title	No. of Courses	Most Common Hours of Credit	Level Offered	Most Common Prerequisite
Probability	12	3	senior	integral calculus
Theory of Functions	12	6	graduate	integral calculus
Differential Geometry	11	3	senior	integral calculus
Advanced Analytic Geometry	10	3	junior	analytic geometry
Series	10	3	graduate	integral calculus
Groups	9	3	graduate	integral calculus
Topology	9	3	graduate	integral calculus
Applied Mathematics	8	3	senior	differential equations
Analysis	8	3	senior	integral calculus
Use of Mathematical Instruments	7	1	freshman	none stated
Determinants and Matrices	6	3	senior	integral calculus
Mechanical Drawing	6	6	freshman	none stated
Advanced Plane Geometry	5	3	junior	none stated
Descriptive Geometry	5	3	sophomore	mechanical drawing
Higher Mathematics for Engineers and Scientists	5	6	graduate	differential equations
Partial Differential Equations	5	3	graduate	differential equations
Calculus of Finite Differences	5	3	senior	differential equations
Integral Equations	5	3	graduate	advanced calculus
Mathematical Logic	5	3	senior	integral calculus
Non-Euclidean Geometry	4	3	senior	integral calculus
Calculus of Variations	3	3	graduate	advanced calculus
Tensor Analysis	3	3	graduate	integral calculus
Foundations of Geometry	3	3	senior	integral calculus
Special Topics in Algebra	3	3	senior	none stated
Algebraic Geometry	2	4	senior	integral calculus
Applications in Geometry	2	3	senior	none stated
Actuarial Mathematics	2	3	senior	differential calculus
Life Contingencies	1	6	graduate	none stated
Mathematics of Life Insurance	1	3	senior	consent of department
Riemannian Geometry	1	3	senior	integral calculus
Least Squares	1	2	senior	integral calculus
Curve Fitting	1	1	senior	integral calculus
Quality Control	1	3	senior	statistics
Set Theory	1	2	graduate	none stated
Theory of Fields	1	2	graduate	higher algebra
Linear Algebras	1	2	graduate	none stated
Theory of Ideals	1	3	graduate	none stated
Boolean Algebra	1	3	senior	integral calculus

Dimension Theory	1	3	graduate	advanced calculus
Theory of Relativity	1	3	senior	consent of department
Lattice Theory	1	3	graduate	higher algebra
Boundary Value	1	3	senior	differential equations
Circuit Analysis	1	3	senior	none stated
Topics in Biometry	1	1	senior	college algebra

SUMMARY

If we assume our sample is representative, in a typical mathematics department about sixteen courses will be offered. Usually the most elementary course will be intermediate algebra. Other courses will be college algebra, plane trigonometry, analytic geometry, two courses in calculus, differential equations, theory of equations, college geometry, statistics, mathematics of finance, and advanced calculus. If the school offers a teacher training program a course in the teaching of mathematics is usually included. If graduate work is offered courses in modern algebra, complex variables and vector analysis are the most likely to be included. The most usual requirements for a mathematic major are twenty-four hours of mathematics with at least one year of calculus.

There are drawbacks to this survey. If the labor were not prohibitive, one might wish to study all schools rather than a ten percent sample. However this at least may give some idea of the present situation. It is granted that fault may be found with this study. For example: not all catalogues state the full content of the courses described; and one is not sure how how closely the actual content agrees with the catalogue description. In many cases it was felt that there were prerequisites to courses which were not indicated. More exact information might be obtained if finances permitted a personal visit to the actual schools in question.

CHEMICAL PROGRESS WEEK

The second Chemical Progress Week has been scheduled for May 16–21, 1955, it was announced by the Manufacturing Chemists' Association, sponsor of the event.

"The purpose of the week," said William C. Foster, MCA president, "is to bring home to the American people the significance of the chemical industry in terms of their daily life. Last year we feel we got off to a good start. This year we hope to do even better."

The board of directors of the MCA, made up of top-level industry executives, will serve as the sponsoring committee. The program will be national in scope with emphasis on local-level activities, particularly in chemical industry plant-communities, the announcement said.

The Manufacturing Chemists' Association represents over 90% of the productive capacity of the chemical industry.

PHOTOGRAPHIC TIME AND APERTURE CORRECTIONS AT FINITE WORKING DISTANCES

RONALD L. IVES
Cornell Aeronautical Laboratory, Inc.

INTRODUCTION

When photographs are taken at small working distances, it is necessary to extend the bellows of the camera to obtain sharp focus. When the working distance is extremely small, such as ten times the focal length of the lens or less, trouble is experienced with underexposures unless a correction is made in the exposure time.

This correction, which is determined by the fundamental dimensions of the camera system, is relatively simple and straightforward, but has recurrently caused trouble to students, teachers, and even professional photographers.

FUNDAMENTAL RELATIONS

The numerical aperture of a lens, commonly referred to as the "*F*" number, is determined by dividing the focal length of the lens by the diameter of the stop in use. This may be stated as:

$$F = \frac{f}{d} \tag{1}$$

in which

F = numerical aperture of lens
f = focal length of lens
d = diameter of stop opening

Note that F is nondimensional and is independent of the system of units used, provided the linear measurements are all made in the same system of units.

Focal length of a lens is commonly defined as the distance between center of the lens and the image of a point at infinity when the image has minimum diameter. Points at infinity being somewhat hard to locate, most hand cameras are focused at points about 300 times the focal length distant from the lens. Astronomical and surveying cameras of long focal length are focused on more distant objects by the method of parallax, the ray structures about the crater Copernicus on the Moon being suitable "targets."

WORKING RELATIONS

When the object distance, which is the distance between the lens

and the object to be photographed, is only a few focal lengths, the camera bellows must be extended to secure a sharp image, in accord with the general lens law, which states:

$$\frac{1}{U}+\frac{1}{V}=\frac{1}{f}. \tag{2}$$

In the above formula,

U = distance between object and lens

V = distance between image and lens

f = focal length of the lens, with object, lens, and image being arranged as in Figure 1.

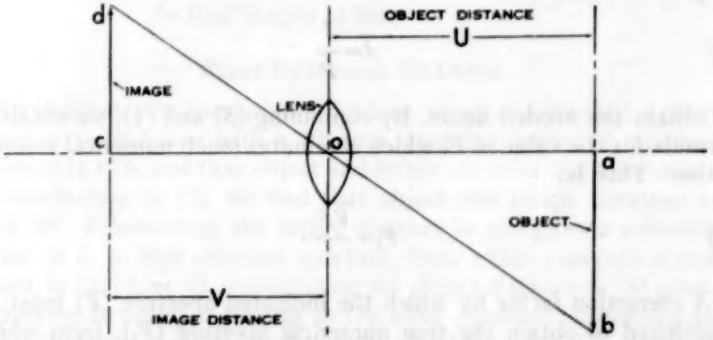


FIG. 1.—Fundamental relations of object, lens, and image.

When the object is many focal lengths from the lens, the term U above is very large relative to V and f, and its reciprocal, consequently, is small, so that it may usually be disregarded entirely. If, however, the object is quite close to the lens, the term $1/U$ is no longer negligibly small, and the F number of the stop, computed from the focal length of the lens, is no longer correct.

CORRECTION PROCEDURE

With sharp focus maintained, an increase in the distance between lens and image, all other factors remaining the same, actually means that the light entering the camera is spread over a larger area. This leads to underexposure if the F numbers engraved on the lens mount are used, uncorrected, to compute exposure. Under these conditions the F index merely indicates the aperture in terms of the focal length of the lens. What we need, for correct exposure computation, is the numerical aperture, F_1, obtained by dividing the image distance V of (2) by the diameter of the aperture. This true numerical aperture, F_1, is found from:

$$F_1 = \frac{V}{d} \tag{3}$$

where

F_1 = true numerical aperture
V = distance from lens to image
d = diameter of aperture

Unfortunately we do not know the actual diameter of the aperture, and its measurement requires disassembly of the lens mount, a time-consuming procedure. We have, however, in (1), a definition of this aperture diameter in terms of the focal length of the lens, and we may rewrite (1) as:

$$d = \frac{f}{F} \tag{4}$$

to obtain the needed figure. By combining (3) and (4), we obtain a formula for the value of F_1 which eliminates much numerical computation. This is:

$$F_1 = \frac{VF}{f}. \tag{5}$$

A correction factor by which the indicated aperture (F) must be multiplied to obtain the true numerical aperture (F_1), from which exposure is computed, is obtained by dividing (5) through by F, giving:

$$\text{Stop correction factor} = \frac{F_1}{F} = \frac{V}{f}. \tag{6}$$

V for this equation is determined by direct measurement (use a ruler), and f is usually engraved on the lens mount, but can also be determined by direct measurement of the distance from lens to image when a distant object is in sharp focus.

In many instances, because of the shape and bulk of the camera, it is quite difficult to measure the image distance (V), but easy to measure the object distance (U). Also, as the object distance is greater than the image distance for most photographic setups, the measurement can be made to a higher percentage of accuracy, all other factors remaining the same.

To determine the correction factor in terms of object distance (U) and focal length of the lens (f), equation (2) is solved for V:

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{f} \tag{2}$$

$$Vf+Uf=UV$$

$$V(U-f)=Uf$$

$$V=\frac{Uf}{U-f}. \tag{7}$$

Substituting this value of V in equation (6) gives:

$$\text{Stop correction factor}=\frac{F_1}{F}=\frac{U}{U-f}. \tag{8}$$

In which, as in previous equations:

U = distance from center of lens to object
f = focal length of lens

First Practical Example

To illustrate the correction procedure, let us assume that a photograph is to be taken with a 10″ lens, that the indicated numerical aperture is F16, and that object and image are to be the same size.

Substituting in (2), we find that object and image distances are each 20″. Substituting the image distance in (6) gives a correction factor of 2, so that effective aperture, from which exposure is computed, is 2×16, or 32. Substituting the object distance in (8) gives a correction factor of 2, leading to the same exposure. These two values furnish a mutual check, but only one of them need be computed to determine the correction factor.

Exposure Time Corrections

With some exposure meters and exposure time computers, it is not convenient to correct the indicated stop number when the camera bellows is extended; and increase of exposure time is the most convenient method of compensating for increased image distance.

For any illuminated object, the photographic exposure time, all nonconsidered factors remaining the same, can be stated:

$$T\cong\frac{1}{\frac{\pi}{4}d^2}. \tag{9}$$

In which

T = exposure time in seconds
d = diameter of lens aperture

Here again, we will have difficulty in measuring d directly, but we can determine it, in terms of the focal length of the lens and the nu-

merical aperture from (4). Substituting this value in (9), we obtain:

$$T = kF^2. \tag{10}$$

In which

k = a compound constant including film speed, focal length of lens, and proportionality factors determined by the system of measurement in use

F = numerical aperture of lens

As we have seen previously, when the object is not at photographic infinity, the image distance is some figure greater than the focal length of the lens. In consequence, the light incoming through the lens is spread over a larger area, and the indicated aperture number must be increased proportionally in order to compute exposure correctly. This is most conveniently done by correcting the indicated numerical aperture F, in accord with (6) or (8), so that equation (10) becomes:

$$T_1 = KF_1^2. \tag{11}$$

In which

T_1 = corrected exposure time

F_1 = corrected numerical aperture.

To determine a time correction factor by which T, the exposure time for an object at infinity, must be multiplied to give the correct exposure time, T_1, for an object at a small finite distance, we divide (10) by (11), obtaining:

$$\text{Time correction factor} = \frac{T_1}{T} = \frac{F_1^2}{F^2}. \tag{12}$$

Then, we may evaluate this factor in terms of the image distance from (6), obtaining:

$$\text{Time correction factor} = \frac{T_1}{T} = \left(\frac{V}{f}\right)^2 \tag{13}$$

or, similarly, in terms of the object distance:

$$\text{Time correction factor} = \frac{T_1}{T} = \left(\frac{U}{U-f}\right)^2. \tag{14}$$

Second Practical Example

Using the same conditions as in the first practical example—namely a 10″ lens, an indicated numerical aperture of F16, and object distance equal to image distance (both 20″ in this case), we can deter-

mine the factor by which the indicated exposure time (shown by exposure meter or computer) must be multiplied to give correct exposure by substitution.

Known that the image distance is 20″, we may substitute in (13), obtaining a time correction factor of $(20/10)^2$, or 4. If we prefer to use the object distance (here 20″), we may substitute in (14), obtaining a time correction factor of $[20/(20-10)]^2$, or 4. Note that the correction factor is the same whether computed from object distance or image distance. This factor tells us that the exposure time computed from the indicated aperture must be multiplied by 4 to give a correctly exposed negative for this particular arrangement of object, lens, and image.

Magnification

At the instant when a photograph is taken, ratio of object size to image size, called the magnification, is exactly the same as the ratio of object distance to image distance, as can be determined from Figure 1. Here, triangles *oab* and *ocd* are similar. Hence:

$$\frac{oa}{ab}=\frac{oc}{cd} \quad \text{and}$$

(15)

$$\frac{cd}{ab}=\frac{oc}{oa}.$$

Now, the term cd/ab is the ratio of the image size to the object size, or the magnification; and the term oc/oa is the ratio of the image distance to the object distance, or V/U (by inspection from Figure 1).

Remembering that the magnification, M, is equal to V/U, we may manipulate (2) by standard algebraic processes to secure a formula for the bellows extension (image distance) for any given magnification, and for the time correction needed at that magnification, as follows:

(2)
$$\frac{1}{U}+\frac{1}{V}=\frac{1}{f}$$

$$Vf+UF=VU$$

$$\left[M=\frac{V}{U}\right]$$

(16)
$$MF+f=V$$

$$V=(M+1)f$$

and, as the time correction factor for a given bellows extension is

$(V/f)^2$, from (13), we may express this in terms of magnification by dividing (16) through by f. The time correction for a magnified image then becomes:

$$\frac{T_1}{T}=\left(\frac{V}{f}\right)^2=(M+1)^2 \tag{17}$$

and the exposure time indicated by the exposure meter or computer must be multiplied by this figure in order to secure a satisfactory negative.

Similarly, the stop correction factor is shown to be V/f in (6). This may be expressed in terms of magnification by dividing (16) through by f, the result being:

$$\frac{F_1}{F}=\frac{V}{f}=(M+1). \tag{18}$$

When magnification is known, the indicated stop can be multiplied by this factor before exposure is computed, to compensate for bellows extension.

TABLE I

V	F_1/F	t_1/t
1.0	1.0	1.0
1.1	1.1	1.21
1.2	1.2	1.44
1.3	1.3	1.69
1.4	1.4	1.96
1.5	1.5	2.25
1.6	1.6	2.56
1.7	1.7	2.89
1.8	1.8	3.24
1.9	1.9	3.61
2.0	2.0	4.00
2.25	2.25	5.07
2.5	2.5	6.25
2.75	2.75	7.34
3.0	3.0	9.0
4.0	4.0	16.0
5.0	5.0	25.0
6.0	6.0	36.0
7.0	7.0	49.0
8.0	8.0	64.0
9.0	9.0	81.0
10.0	10.0	100.0

UNIVERSAL CORRECTION TABLES

The foregoing computations and equations permit us to correct

the indicated aperture for any errors introduced by bellows extension and give correction formulae for use with indicated aperture, computed time, or known magnification. If we express all distances in terms of multiples of the focal length of the lens in use, it is possible to compute universal correction tables, having wide utility.

In Table I, V, the image distance, is expressed in focal lengths; the factor F_1/F from (6) is that by which the indicated aperture must be multiplied to compensate for bellows extension in order to compute exposure correctly, and the factor t_1/t from (13) is that by which the computed exposure time, based on indicated aperture, must be multiplied to compensate for bellows extension. Note specifically that *either* factor may be used to correct for bellows extension, but that *only one* correction is necessary.

TABLE II

U	F_1/F	t_1/t
1.0	α	α
1.1	11.0	121.0
1.2	6.0	36.0
1.3	4.3	18.5
1.4	3.5	12.25
1.5	3.0	9.0
1.6	2.6	6.76
1.7	2.43	5.90
1.8	2.25	5.06
1.9	2.11	4.45
2.0	2.0	4.0
2.25	1.8	3.24
2.50	1.66	2.77
2.75	1.56	2.43
3.0	1.5	2.25
4.0	1.33	1.77
5.0	1.25	1.56
6.0	1.2	1.44
7.0	1.17	1.37
8.0	1.14	1.30
9.0	1.125	1.265
10.0	1.11	1.23
12.0	1.09	1.18
15.0	1.07	1.15
20.0	1.05	1.10
40.0	1.026	1.05
50.0	1.02	1.04
60.0	1.017	1.034
70.0	1.014	1.028
80.0	1.013	1.023
90.0	1.011	1.022
100.0	1.01	1.02

In Table II, U, the object distance, is expressed in focal lengths; the factor F_1/F from (8) is that by which the indicated aperture must be multiplied to compensate for bellows extension in order to compute exposure correctly, and the factor t_1/t from (14) is that by which the computed exposure time, based on indicated aperture, must be multiplied to compensate for bellows extension. Here again, *either* factor may be used to correct for bellows extension, but *only one* correction is necessary.

In Table III, the magnification, M, which is evaluated from V/U, is given as a decimal fraction, and the value of the image distance, V, is expressed in focal lengths (from (16)). U is evaluated in magnifications by substituting the value of V from (16) into (2) as follows:

(16) $$V=(M+1)f$$

$$\frac{1}{U}+\frac{1}{(MH)f}=\frac{1}{f}$$

$$(M+1)f+U=(M+1)U$$

$$U=\left(\frac{M+1}{M}\right)f$$

or, if U is expressed in focal lengths:

(19) $$U=\frac{M+1}{M}.$$

TABLE III

M	F_1/F	t_1/t	U	V
.1	1.1	1.21	11.0	1.1
.2	1.2	1.44	6.0	1.2
.3	1.3	1.69	4.3	1.3
.4	1.4	1.96	3.5	1.4
.5	1.5	2.25	3.0	1.5
.6	1.6	2.56	2.66	1.6
.7	1.7	2.89	2.43	1.7
.8	1.8	3.24	2.25	1.8
.9	1.9	3.61	2.11	1.9
1.0	2.0	4.0	2.0	2.0
2.0	3.0	9.0	1.5	3.0
3.0	4.0	16.0	1.33	4.0
4.0	5.0	25.0	1.25	5.0
5.0	6.0	36.0	1.20	6.0
6.0	7.0	49.0	1.16	7.0
7.0	8.0	64.0	1.14	8.0
8.0	9.0	81.0	1.12	9.0
9.0	10.0	100.0	1.11	10.0
10.0	11.0	121.0	1.1	11.0

The aperture correction factor F_1/F, previously defined, is derived from V/f (as in (6)), and the time correction factor, t_1/t, from (16).

By multiplying the factors U and V of these tables by the focal length of the lens is use, aperture and time correction tables for any specific lens can be constructed in a few minutes. Care must be taken to use compatible units, because a table prepared by multiplying U and V by the focal length of the lens in millimeters will not give very satisfactory results if the other measurements are made in inches.

Summary

Photographic exposure for objects at a finite distance from the camera is influenced by the distance between the object and the lens. When the object-lens distance is more than about 30 focal lengths of the lens in use, it can be considered as at photographic infinity, and no correction of stop or exposure time is either needed or desirable in ordinary exposure computation, such as is done with a Weston, or similar exposure meter.

When the object in less than about 30 focal lengths from the lens, correction of either numerical aperture value or exposure time is desirable, the actual aperture value being double the indicated aperture value when object and image distance are equal, at which setting the magnification is 1, the aperture correction value is 2, and the time correction value is 4.

As magnification increases, so do the corrections, so that at a magnification of 10, the actual numerical aperture is 11 times the indicated value and the time correction factor becomes 121. Although film latitude will usually permit a variation in exposure by a factor of 2, failure to correct for bellows extension at magnifications greater than 1 will almost invariably make retakes necessary.

Data here given show the values of the various corrections for a wide range of image distance, object distance, and magnification values, as well as the sources of the correction factors, so that these factors, for special cases not included in the tables, can be computed readily. Rather fortunately, there is an excellent concordance between formula values for these corrections and "values that work."

RESEARCH GRANT AWARDED ADELPHI

Charles Pfizer & Co., Inc. of Brooklyn has awarded a $2,200 research grant to Adelphi College, Garden City.

The announcement was made today by Jasper V. Kane of Garden City, vice president in charge of research for Pfizer, one of the nation's largest chemical and drug concerns, a pioneer in the field of antibiotics.

Dr. Theodore Bieber of Kew Gardens, assistant professor of Chemistry at the college, will use the grant for biochemical research on new methods of amide synthesis.

EFFECTIVE HIGH SCHOOL RECRUITING

R. E. Dunbar and F. H. Sands
North Dakota Agricultural College, Fargo, North Dakota

It is a well known fact among educators and industrialists that the supply of available young chemists is drying up at the source (1, 2). Wholesome publicity (3), open houses (4), personal interviews, radio and television programs, and other similar activities have been helpful in many instances in encouraging young men and women to enter the chemical profession. Our own department three years ago inaugurated a new approach that has already produced sufficiently encouraging results to warrant elaboration and perhaps repetition by other institutions of higher learning.

Our plan has been to organize, prepare and present a complete chem-MYSTERY show before a large number of representative high schools in the area each year. Demonstrations have been selected that are somewhat spectacular but substantial and instructive. Staff members have been assigned to the presentations who are convincing speakers and effective demonstrators. A few minutes are utilized at the beginning of each appearance to vividly point out the opportunities and personal satisfaction to be gained from a career in the profession of chemistry. Available vocational guidance bulletins are distributed to students and staff in the several schools visited. We have been selling the merits of chemistry rather than those of any particular institution with the belief that all schools in the area will then get their share of promising students.

We have deliberately avoided the spectacular for its own sake and to the exclusion of related discussion for we have sought to inspire rather than mystify. We have invited questions from the audience. Chemical explanations have been readily given where applicable. Obviously considerable advance correspondence with school administrators is required to satisfactorily arrange the necessary details for a successful show.

Among numerous demonstrations that have recently been used, the following have been found to be particularly satisfactory and effective. Likewise, these demonstrations require a minimum of equipment, no gas or running water, and can therefore, be readily presented in the usual high school class room or auditorium. The necessary patter, humor, explanation, or combination of the above can usually be supplied by the adult demonstrator, depending upon the age group or occasion. Adequate details and directions are given for each experiment so that they can easily be repeated by other interested chemists.

Suggested Demonstrations

Fire on Ice. A few small lumps of calcium carbide on ice will quickly generate enough acetylene to be easily burned. A small lump of metallic potassium, simultaneously added, will make the acetylene self igniting.

Chemical Volcano. A small crucible, placed on a large asbestos mat, and filled with dry ammonium dichromate makes a very effective potential volcano. Ignite with a strip of magnesium ribbon.

Red, White and Blue. A weak solution of ferric chloride is placed in a large transparent container. When this solution is poured into three beakers containing respectively, a small amount of ammonium thiocyanate, silver nitrate and potassium ferrocyanide solutions, the traditional patriotic colors are produced.

Solid Alcohol. When 85 ml. of ethanol is added quickly, or with stirring, to 15 ml. saturated calcium acetate solution the resulting mixture readily solidifies into a firm gel. This product may be burned readily. If placed on red chromic oxide it ignites spontaneously.

Synthetic Rubber. A 54.3% solution of Chemigum (available from Goodyear Chemical Division, Akron, Ohio), when diluted with five parts of water, is readily coagulated by the addition, with stirring, of dilute hydrochloric acid. The effect is spectacular.

Dry Ice Engine. A few ml. of alcohol or other low freezing organic solvent are added to a toy steam engine. The subsequent addition of a few small pieces of dry ice will generate sufficient CO_2 pressure to operate the engine.

Mercury Hammer. A mercury hammer, that will drive nails, can be prepared by placing a small paper container, filled with mercury, in a beaker. Add a suitable handle and cover the mercury with alcohol or other low-freezing organic solvent. The addition of a generous supply of dry ice will soon completely freeze the mercury.

Bouncing Putty. Bouncing Putty (available from General Electric Co., Schenectady, N. Y.), offers fascinating possibilities.

Plastic Foam. The production of plastic foam is unusual and spectacular. Complete kits are available at reasonable prices from Chemical Department, General Electric Co., Pittsfield, Mass., or Bakelite Corp., 30 East 42nd St., New York 17, N. Y.

Pharaoh's Serpents. If equivalent amounts of mercuric nitrate and potassium thiocyanate, both in solution, are mixed a copious precipitate of mercuric thiocyanate forms. This precipitate should be filtered and, while moist, formed into small cones. When dry they may be ignited with the usual effect.

Iodine Clock. Two solutions are prepared, the first containing approximately 6 g. potassium iodate in 32 oz. of water; and the second containing approximately 3 g. sodium sulfite, a little starch paste

and about 20 drops of concentrated sulfuric acid, all in 32 oz. of water. When equal amounts of the two solutions are mixed the usual time reaction occurs with the production of the sudden dark color. The use of more acid speeds the time of the reaction, and vice versa. If the dark reaction mixture is then poured into a small amount of household laundry bleach, the color just as suddenly disappears.

Water, Wine and Milk. A large container is provided, filled with water and a small amount of phenolphthalein. Five beakers are arranged, the first of which contains a few ml. (excess) concentrated sulfuric acid, the second and fourth a lesser amount of sodium hydroxide solution, the third and fifth are empty. When the phenolphthalein solution is poured into each beaker in turn, the second and fourth develop the traditional red color, the remainder are clear. If the contents of each beaker are now returned to the large container, the one with the sulfuric acid first, the color again disappears. A milk bottle containing a little saturated antimony trichloride in hydrochloric acid, may now be filled and develops a milk-like appearance.

Hydrogen and Oxygen. Hydrogen soap bubbles will float in air. The addition of oxygen produces explosive bubbles. This makes a good finale.

Phosphine Rings. If phosphine gas, readily generated from calcium phosphide and water, is bubbled through water into still air, it produces spectacular smoke rings.

Bologna Bottles. A new item recently made available by the W. M. Welch Manufacturing Co., Chicago, Ill., demonstrates nicely the strength and stress of glass.

Crude Oil. Representative samples of North Dakota crude oil or other local chemical products always attracts favorable attention.

Chameleon Colors. Litmus paper dipped into blue copper sulfate solution turns red, but in red sodium hydroxide-phenolphthalein solution turns blue. Similar effects may be obtained with numerous indicators or colored reaction solutions, and designs painted on paper or treatment of plain pictures to produce various flags, blushing movie stars, etc.

Foam. A mixture of aluminum sulfate solution, and sodium bicarbonate solution containing a small amount of emulsifying agent, produces a copious foam. Commercial Foam Type chemicals are available from The Safety Fire Extinguisher Co., New York 1, N. Y., and others. "Dow Corning Antifoam, A Spray" may be used to dissipate such foams in a relatively short period.

Pyrophoric Iron and Lead. A good pyrophoric powder is easily made by heating ferrous oxalate or lead tartrate in a hard glass test tube until no more fumes are evolved. The tube is then tightly corked

and allowed to cool. When sprinkled into air, the colloidal particles of metal and carbon take fire spontaneously.

Bibliography

1. Neckers, J. W., *Chem. Eng. News.*, 30, 2294 (1952).
2. Hildebrand, J. H., *Chem. Eng. News.*, 31, 5084 (1953).
3. North Dakota Agricultural College, "Chemistry and You," Fargo, North Dakota, 1953.
4. Dunbar, R. E., *J. Chem. Educ.*, 5, 531 (1928).

PROBLEM DEPARTMENT

Conducted by Margaret F. Willerding

Harris Teachers College, St. Louis, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, Harris Teachers College, St. Louis, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

In general when several solutions are correct, the ones submitted in the best form will be used.

LATE SOLUTIONS

2400. *C. W. Trigg, Los Angeles, Calif.*

2410. *C. W. Trigg, Los Angeles, Calif.*

2411. *C. W. Trigg, Los Angeles, Calif.*

2417. *J. W. Lindsey, Amarillo, Tex.*

Editor's Note: In the March 1954 issue of The Problem Section there appeared two problems numbered 2402. The solution of 2402 (proposed by C. W. Trigg, Los Angeles City College) was omitted from the October issue. This solution appears below.

2404. *Proposed by C. W. Trigg, Los Angeles City College.*

Show how the surface of a cube may be divided by one continuous cut into two parts which may be folded into congruent hexahedra having congruent triangular faces.

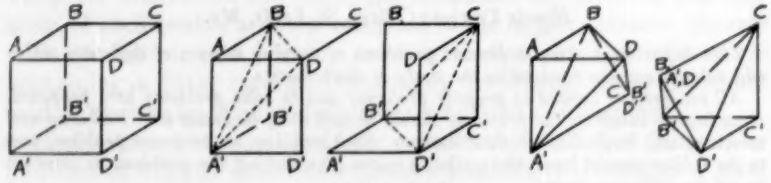
Solution by the proposer

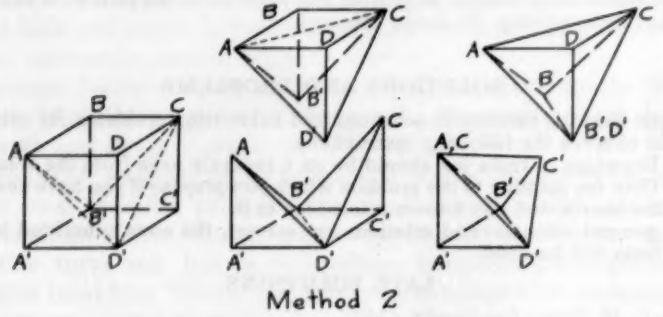
Let the vertices of one face of a cube with edge x be A, B, C, D, in order and the orthogonal projections of these vertices on the opposite face be A', B', C', D'.

Make the cut along $BB'A'D'DCB$. Fold in along DB, BA', $A'D$, CB', $B'D'$ and $D'C$. Each face of the cube will thereby be folded into two isosceles right triangles. When C, B', and D', and B, A', and D come into coincidence two hexahedra with congruent right angular faces are formed. The cut is a 6-segment cut of length $6x$, and six creases are required.

In a second and more expeditious method the cut is made along $B'CD'AB'$. Now fold in along AC until B' and D' coincide. Similarly, fold in along $B'D'$ until A and C coincide. The cut is a 4-segment cut of length $4\times 2x$, and two creases are required.

Since each hexahedron is composed of two trirectangular tetrahedra, the hexahedra are congruent and the volume of each one is $2(1/3)(1/2)x^3$ or one-third the volume of the generating cube.





2419. *Proposed by Felix John, Eddington, Pa.*

In circle O, P is the midpoint of chord AB. Chords RS and TV pass through P. RV cuts AP at M, and ST cuts PB at N. Prove, by high school geometry, if possible, that $MP=PN$.

Solution by Arthur Eilberg, Philadelphia, Pa.

Draw $TY \| AB$.

$$\angle PMR \text{ m } 1/2(AV+BT+TR)$$
$$\angle PYR = \angle 2+\angle TYR$$
$$\angle PYR \text{ m } 1/2(AV+BT+TR)$$
$$\therefore \quad \angle PMR=\angle RYR$$

and $MPRY$ can be inscribed in a circle.

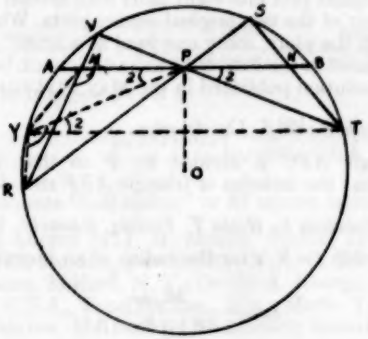
$\angle NTP = \angle MRP$ ($\measuredangle 1/2\overset{\frown}{VS}$)

$\angle MYP = \angle MRP$ ($\measuredangle 1/2\overset{\frown}{MP}$)

$\therefore \angle NTP = \angle MYP$

$\therefore \triangle MYP \cong \triangle PTN$ ($PY = PT$; $\angle 2 = \angle 2$; $\angle MYP = \angle NTP$)

$\therefore MP = PN$



2420. *Proposed by C. W. Trigg, Los Angeles City College.*

Show that if the condition $a^2+b^2+c^2 = 2(ab+bc+ca)$ exists between the sides of a triangle, then its incircle and its three excircles can be arranged in a configuration such that each circle is tangent to the other three, not all at the same point.

Solution by the proposer

The radius of the incircle is $r = 2\Delta/(a+b+c)$, and the radii of the excircles are $r_a = 2\Delta/(b+c-a)$, $r_b = 2\Delta/(c+a-b)$, $r_c = 2\Delta/(a+b-c)$, where Δ is the area of the triangle.

It was shown in problem 2293, SCHOOL SCIENCE AND MATHEMATICS, 53, 75, January 1953, that the radius of the circle internally tangent to three externally tangent circles, with radii r_a, r_b, r_c, is given by

$$r = r_a r_b r_c / [2\sqrt{r_a r_b r_c (r_a + r_b + r_c)} + (r_a r_b + r_b r_c + r_c r_a)].$$

When this equation is cleared of fractions and radicals, we have

$$r_a^2 r_b^2 r_c^2 + r_a^2 r_b^2 r^2 + r_b^2 r_c^2 r^2 + r_c^2 r_a^2 r^2 = 2 r_a r_b r_c r (r_a r_b + r_a r_c + r_a r + r_b r_c + r_b r + r_c r).$$

In this equation, which is symmetrical in r_a, r_b, r_c, r, we may substitute in any order the values of the radii as given in the first paragraph, and after simplification obtain

$$a^2+b^2+c^2 = 2(ab+bc+ca).$$

2421. *Proposed by C. W. Trigg, Los Angeles City College.*

Solve the equation:

$$\prod_{m=1}^{n} \cos mx = 1.$$

Solution by the proposer

If $|c_i| \leq 1$ and

$$N = \prod_{i=1}^{k} c_i,$$

then $|N|<1$ if any $|c_i|<1$. Since $|\cos y|\leq 1$, it follows that $|\cos mx|=1$. Therefore, if n is of the form $4q$ or $4q+3$, $x=k\pi$, $k=0, 1, 2, \cdots$; if n is of the form $4q+1$ or $4q+2$, $x=2k\pi$, $k=0, 1, 2, \cdots$.

2422. *Proposed by Leon Bankoff, Los Angeles.*

"A realtor exchanged two square plots with integer sides and of different areas for one rectangular plot equal to the combined areas of the other two. He then subdivided the rectangular plot into eight plots with integer sides, each of which was smaller than either of the two original square plots. What are the least possible dimensions of all the plots, using one yard as a unit?"

This problem appeared in the Feb. 1953 issue of SCHOOL SCIENCE AND MATHEMATICS as 2346 with solution published in the May 1953 issue.

2423. *Proposed by Leon Bankoff, Los Angeles.*

Side BC of triangle ABC is divided by P so that $BP/PC=(BA-AP)/(AP-AC)$. Show that the incircles of triangle ABP and ACP are equal.

Solution by Herta T. Freitag, Roanoke, Va.

Using the relationship $r=K/s$ for the radius of an incircle,

$$r_{ABP}=\frac{2K_{ABP}}{BP+AP+AB}$$

and

$$r_{APC}=\frac{2K_{APC}}{PC+AP+AC}.$$

But

$$\frac{K_{ABP}}{K_{APC}}=\frac{BP}{PC}\ \text{(the triangles have equal heights)}$$

and therefore:

$$\frac{r_{ABP}}{r_{APC}}=\frac{BP(PC+AP+AC)}{PC(BP+AP+AB)}$$

$$=\frac{BP\cdot PC+BP(AP+AC)}{BP\cdot PC+PC(AP+AB)}.$$

However:

$$BP(AP+AC)=PC(BA+AP),\ \text{(given)},$$

and consequently $r_{ABP}=r_{APC}$, and the two incircles of triangles ABP and ACP are equal.

Solutions were also offered by Richard H. Bates, Milford, N. Y.; A. H. Haynes, Tacoma, Wash.; Brother Felix John, Eddington, Pa.; Walter T. Grant, Rutherford, N. J.; C. W. Trigg, Los Angeles, Calif.; and the proposer.

2424. *Proposed by Felix John, Philadelphia.*

Find the area of the triangle whose sides are $\sqrt{673}/2$, $\sqrt{505}/2$, and $2\sqrt{37}$.

Solution by Charles H. Butler, Kalamazoo, Michigan

To express the lengths of all sides in similar form, note that

$$2\sqrt{37}=\frac{\sqrt{592}}{2}.$$

Now consider the given lengths to be expressed in some unit, say inches. No generality is lost if we re-express them in half-inches, and by doing so we simplify the expressions to $c=\sqrt{673}$, $a=\sqrt{505}$, and $b=\sqrt{592}$ respectively. Then

$$\cos A=\frac{592+673-505}{2\sqrt{(592)(673)}}=\frac{380}{\sqrt{(592(673)}}$$

by cosine law.

$$\sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\frac{(380)^2}{(592)(673)}}=\frac{504}{\sqrt{(592)(673)}}.$$

$$\text{Area of the triangle}=\tfrac{1}{2}bc\sin A$$

$$=\tfrac{1}{2}\sqrt{592}\sqrt{673}\left(\frac{504}{\sqrt{(592)(673)}}\right)=252.$$

Thus the area is 252 square "half-inches" or 63 square inches.

Solutions were also offered by J. H. Means, Austin, Texas; A. R. Haynes, Tacoma, Wash.; Marvin Sandler, Jersey City, N.J.; C. W. Trigg, Los Angeles, Calif.; Richard H. Bates, Milford, N. Y.; Orville A. George, Mason City, Iowa; Sr. Mary Constance, C.S.A., Fond du Lac, Wis.; Herta T. Freitag, Roanoke, Va.; J. Byers King, Denton, Md.; and J. W. Lindsey, Amarillo, Tex.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

For this issue the Honor Roll appears below.

2422, 23, 24. *John Hewill, Upper Canada College, Toronto.*

2424. *Robert Cooper, Denton Md.; E. David Mellets, Denton, Md.; Charles King, Chestertown, Md.; Edward F. Sidor, Chicago, Ill.; Lee Dresden Goldberg, Hillside, N. J.*

EDITOR'S NOTE: In proposal 2429 which appeared last month, the four bottom lines got skidded to the right, so that the problem has no solution in its present form. Poor proof reading on the new editor's part!

PROBLEMS FOR SOLUTION

2437. *Proposed by C. W. Trigg, Los Angeles, Calif.*

Squares are constructed externally on the legs of a right triangle ABC. The joins of the vertices of the acute angles to the remote vertices of the squares on the opposite legs intersect in M, and cut AC and AB in B' and A' respectively. Show that:

1. The segment of either leg adjacent to the right angle is the mean proportional between the remote segments of the legs.
2. The product of the ratios into which M divides AA' and BB' is equal to the square of the sum of the legs divided by the product of the legs.
3. The areas of the triangles AMB' and $A'MB$ are in the same ratio as the adjacent legs.

2438. *Proposed by Brother Felix John, Eddington, Pa.*

Show that $x^6-x^5y+4x^4y^3-2x^3y^3+4x^2y^4-xy^5+x^6>0$ for all real values of x and y. (From Chrystal's Algebra, Vol. II.)

2439. *Proposed by Dewey C. Duncan, Los Angeles, Calif.*

If $x_1, x_2, \cdots, x_n$ are positive real numbers, not all being equal, then show

$$\frac{x_1+x_2+\cdots+x_n}{n} > \sqrt[n]{x_1x^2\cdots x_n}$$

(a generalization of 2363).

2440. *Proposed by C. W. Trigg, Los Angeles, Calif.*

There are only two five-digit integers formed from two different digits whose square minus one million are permutations of the ten digits.

2441. *Proposed by Nathan Altshiller-Court, University of Okla.*

The product of the powers of a vertex of a triangle with respect to the four tri-tangent circles is equal to the product of the squares of the radii of the four circles.

2442. *Proposed by V. C. Bailey, Evansville, Ind.*

Find an expression for the length of the line parallel to the parallel sides and bisecting the area of a trapezoid in terms of the parallel sides.

BOOKS AND PAMPHLETS RECEIVED

INTRODUCTION TO ATOMIC AND NUCLEAR PHYSICS, Third Edition, Revised and Enlarged, by Henry Semat, Ph.D., *Professor of Physics, The City College, College of the City of New York*. Cloth. Pages xii+561. 15×23 cm. 1954. Rinehart and Company, 232 Madison Avenue, New York 16, N. Y. Price $6.50.

A TREATISE ON ELECTRICITY AND MAGNETISM, Unabridged, Third Edition, by James Clerk Maxwell. Two Volumes Bound as One. Volume I, xxxii+506+11 pages. Volume II, xxiv+500+7 pages. 13.5×20.5 cm. 1954. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $4.95.

TRIGONOMETRY, by Elbridge P. Vance, *Oberlin College*. Cloth. Pages viii+158. 14×21.5 cm. 1954. Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass. Price $3.00.

ANIMAL FORM AND FUNCTION, by W. R. Breneman, *Professor of Zoology, Indiana University*. Cloth. Pages viii+488. 15×23 cm. 1954. Ginn and Company, Statler Building, Boston 17, Mass. Price $6.00.

SCIENCE THE SUPER SLEUTH, by Lynn Poole, *Producer of The Johns Hopkins TV Science Review*. Cloth. 192 pages. 14×20.5 cm. 1954. Whittlesey House, McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $2.75.

THE MICROPHYSICAL WORLD, by William Wilson, Ph.D., *Fellow of King's College, London, and Professor Emeritus of Physics in the University of London*. Cloth. Pages vii+216. 10×16.5 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $3.75.

MODERN SCIENCE AND GOD, by P. J. McLaughlin, D. ès Sc. Cloth. 89 pages. 12×18.5 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $2.75.

CAREERS AND OPPORTUNITIES IN SCIENCE, by Philip Pollack. Cloth. 252 pages. 13×20.5 cm. 1954. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York 10, N. Y. Price $3.75.

UNDERSTANDING NUMBERS: THEIR HISTORY AND USE, by Phillip S. Jones, *Associate Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.* Paper. Pages iii+50. 21.5×28 cm. 1954. Ulrich's Bookstore, 547–549 East University Avenue, Ann Arbor, Mich.

THE PSYCHOLOGY OF INVENTION IN THE MATHEMATICAL FIELD, by Jacques Hadamard. Paper. Pages xiii+145. 13×20.5 cm. 1954. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $1.25.

ARITHMETIC AND CURRICULUM ORGANIZATION. Number 3 in a Series of Monographs on the Teaching of Arithmetic, by Vincent J. Glennon with Catherine Stack and Students. Paper. Pages x+140. 15×23 cm. 1954. Syracuse University, Bureau of School Service, 201 Slocum Hall, Syracuse 10, N. Y. Price $2.00.

EXPERIMENTAL ELECTRICITY FOR THE BEGINNER, Second Edition, by Leonard R. Crow, *Educational Specialist in Design and Development of Training Aids for Teaching Electricity.* Paper. Pages xvi+284. 21×28 cm. 1954. The Scientific Book Publishing Company, Vincennes, Ind.

TEACHERS' MANUAL FOR SCIENCE FOR A BETTER WORLD, by Meister, Keirstead, Shoemaker. Paper. Pages viii+152. 14×19.5 cm. 1954. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, N. Y. Price 96 cents.

OBTAINING AND INTERPRETING TEST SCOPE TRACES, by John F. Rider, *Fellow, Radio Club of America.* Paper. 186 pages. 14×22 cm. 1954. John F. Rider Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $2.40.

PICTURE BOOK OF TV TROUBLES. Volume 1, Horizontal AFC-Oscillator, by John F. Rider Laboratories Staff. Paper. Pages vi+70. 14×22 cm. 1954. John F. Rider Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $1.35.

HOW TO USE TEST PROBES, by Alfred A. Ghirardi and Robert G. Middleton. Paper. Pages iv+172. 14×21.5 cm. 1954. John F. Rider, Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $2.90.

A BUDGET OF PARADOXES, by Augustus De Morgan. Second Edition. Edited by David Eugene Smith. Cloth. Pages 13+387. 13×20.5 cm. 1954. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. Price $4.95.

THE HIDDEN LIFE OF FLOWERS. Photographs by R. H. Noailles, and Translated from the French Text of J. M. Guilcher. Cloth. 93 pages. 13×19 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $4.75.

SCIENCE AWAKENING, by B. L. Van der Waerden. English Translation by Arnold Dresden. Cloth. 306 pages. 17.5×25 cm. 1954. P. Noordhoff, Ltd, Publishers, Groningen, Holland. Price $5.00.

ELEMENTARY STATISTICS, by John M. Howell and Ben K. Gold, *Department of Mathematics, Los Angeles City College, Los Angeles, California.* Paper. Pages v+154. 21×28 cm. 1954. Wm. C. Brown Company, 915 Main Street, Dubuque, Iowa. Price $3.00.

LIVING HEALTHFULLY, by Charles H. Philpott, Ph.D., *Director of Education, St. Louis Public Schools.* Paper. Pages xii+372. 12×19 cm. 1954. Oxford Book Company, 222 Fourth Avenue, New York, N. Y. Price 90 cents, Cloth $1.65.

SCIENCE FACILITIES FOR THE MODERN HIGH SCHOOL, by Paul DeH. Hurd, *Assistant Professor of Education, Stanford University, Stanford, California.* Paper. Pages viii+52. 13.5×23 cm. 1954. Stanford University Press, Stanford, Calif.

KEY-TEST IN CHEMISTRY, by Vinton R. Rawson, *Chairman of Science Department, Senior High School, White Plains, New York.* Paper. Pages x+317+25.

12×19 cm. 1954. Keystone Education Press, Inc., 222 Fourth Avenue, New York, 3, N. Y. Price 75 cents.

ANSWERS TO FUNCTIONAL MATHEMATICS, by Gager, Mahood, Shuster, and Kokomoor. Paper. For Book 1, pages xxix, and Book 2, pages xxxii. 16.5×23 cm. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, N. Y. Price 24 cents each.

MATHEMATICS. A DESCRIPTION OF THE COLLEGE BOARD TESTS IN INTERMEDIATE AND ADVANCED MATHEMATICS. Paper. 28 pages. 16.5×24 cm. 1954. College Entrance Examination Board, c/o Educational Testing Service, P. O. Box 592, Princeton, N. J. Price 50 cents.

SCIENCE. A DESCRIPTION OF THE COLLEGE BOARD TESTS IN BIOLOGY, CHEMISTRY, AND PHYSICS. Paper, 38 pages. 16.5×24 cm. 1954. College Entrance Examination Board, c/o Educational Testing Service, P. O. Box 592, Princeton, N. J. Price 50 cents.

RATIO OF MEN TO WOMAN TEACHERS IN PUBLIC SECONDARY SCHOOLS. A REPORT ON STATUS AND TRENDS, by Ellsworth Tompkins, *Specialist for Large High Schools.* Circular No. 413. November 1954. 14 pages. 20.5×26.5 cm. U. S. Department of Health, Education, and Welfare, Office of Education, Washington 25, D. C.

BIBLIOGRAPHY OF MATERIAL ON ANIMAL EXPERIMENTATION, by the Illinois Society for Medical Research and the National Society for Medical Research Bulletin No. 6. October, 1954. 8 pages. 21.5×27 cm. Illinois Society for Medical Research, 951 East 58th Street, Chicago, 37, Ill.

BOOK REVIEWS

SPACE TRAVEL. AN ILLUSTRATED SURVEY OF ITS PROBLEMS AND PROSPECTS, by Kenneth W. Galland and Anthony M. Kunesch. Cloth. Pages x+205. 14.3×22.5 cm. 1953. Philosophical Library Inc. 15 E. 40th Street, New York City 16, N. Y. $4.75.

Third dimensional travel has been in the news since the flight of the Wright Brothers. With Comic(?) Strips and Science Fiction increasingly featuring this field it is gratifying to have a non-technical yet authoritative book at hand for recourse when the thrillers become over-fantastic.

The authors are rated "technical experts in their own fields." Mr. Galland is also secretary of the British Interplanetary Society. Such official connection should warn the reader to expect his account to be optimistic in outlook; perhaps some may rate it as over-credulous.

The table of contents lists seven chapters. First some history credits the pioneers as well as rocket researches during World War II with basic first efforts. It relates missiles development to the emerging concept of beyond-atmospheric regions of flight. In such contemplation the postulated "Artificial Satellite" was born. From its "gravitationless" vantage point the "Universe Invites."

However, these fanciful space-lures are held in restraint. The partnership of many sciences is invoked to restrict inferential extremes. The solution of the problems of such enterprises are subjected to critically evaluated extrapolations from what is already basic knowledge. Even so the final chapter, "The Stars Beckon," is generously phrased with highly speculative possibilities that may be anticipated when space enthusiasts find that our solar region no longer intrigues.

Over one hundred illustrations are tabled in a two page listing and are explicitly interpreted in an eight page appendix. Twenty-five figures (line drawings) carry interpretative legends below the diagrams. A three page bibliography and a five page double columned index lend aid to the serious student.

Here, space fans, you will find a stabilizing ballast for your super-active imaginations!

B. CLIFFORD HENDRICKS,
457 24th Ave., Longview, Wash.

LIVING CRAFTS, by G. Bernard Hughes. 192 pages. 1954. Philosophical Library, N. Y. Illus. $4.75.

In an age where most things are machine made on an assembly line and the individual workman has little chance to express himself, this book is quite a revelation. It describes many crafts still practiced with simple materials, old methods and tools not powered by electricity. Most of the crafts of course are prevalent in England and have been handed down through the centuries. The historical background of each craft is given with sketches of the old methods and detailed photographs of the present. Included in the crafts are parchment making, soap boiling, glass blowing, horning, pewtering, wood turning, clay pipe making, and several others that the average man has been led to believe went out with the horse and buggy days.

The author describes the simple tools and methods in such an interesting manner and detail that a reader with sufficient inspiration might develop a new hobby based on the information supplied. In addition to being a source of pleasant reading, this book might well be used as a reference for arts, crafts and diverse history and science classes in either high schools or colleges.

JOHN D. WOOLEVER
Mumford High School Detroit, Mich.

SUPERJET, by Lewis Zarem and Ray Cantwell. 126 pages. E. P. Dutton & Co., Inc. N. Y. 1954. $2.50.

This teen-age adventure might have taken place here in our time when jets, superfuels and intrigue are the general run of both fiction and nonfiction. It is directed at the average young man of this decade who usually has some knowledge of flight and is interested in air power. The general plot is not unusual as it revolves about the secret development and testing of a fuel powerful enough to propel a jet plane around the earth non-stop.

The heroes are beset by unknown assailants of an enemy power, at every turn, during supposed secret tests and experimental flights. Eventually, the villains and instigators of their problems are detected and apprehended. The secret formula is saved and obviously a success for future use in our air forces.

There is no objectionable material in the story and several plausible sections with accurate descriptions, give it a true to life feeling. The latter is probably due to both of the authors' technical experiences in flight research.

It is a novel that is easy to read, aimed at a specific reading age group. It could be adapted for use in a radio, television or movie short story with little effort. Although it may appear on countless school library shelves, it wouldn't add anything unusual to a science library shelf.

JOHN D. WOOLEVER

MAN, ROCKETS AND SPACE, by Captain Burr W. Leyson. 188 pages. E. P. Dutton & Co., Inc. N. Y. 1954. $3.50.

This is another excellent book by an author who has written a series of books that contribute to a greater understanding of inventions and phenomena of science for both students and the average layman. It is clearly written, very well illustrated and up to the minute on rocket development and space research. It clarifies and coordinates stories and ideas that have frequently appeared in popular magazines and newspapers, on the subject.

The origin and formation of the earth is the introductory subject, followed by a detailed description of how a rocket works. This subject is then expanded with an explanation of the research being carried on with rockets. Many of the intricate problems in present rocket research are explained as well as cosmic radi-

ation, for the special benefit of those who wonder why rockets may not be used at present in ordinary long distance travel and immediate exploration of other planets.

One of the originators of Germany's V-2 rockets gives detailed plans for space travel and the conquest of space in the future. As a provocative conclusion, Captain Leyson discusses the problems man will face in traveling through outer space, and what his possible destinations may be like after he successfully proceeds beyond the present flight limits.

The amount of material covered in this little book is amazing, as it flows smoothly from one intriguing subject to another. An index and glossary of terms is included even though it is neither a text nor a technical manual.

Captain Leyson has the rare ability to make a technical and difficult subject pleasant to read and easy to understand with no loss in accuracy or reader interest.

JOHN D. WOOLEVER

MODERN CHEMISTRY, by Charles E. Dull. Rev. by William D. Brooks and H. Clark Metcalfe. 587 pages. Henry Holt and Co., N. Y. 1954. $3.88.

This new text was revised and rewritten by two science teachers, the original of which was written by the late Charles E. Dull. It is aimed at the high school chemistry curriculum with particular areas designed for superior students, thus serving a group sometimes neglected. Although there is more material in the book than could possibly be covered in one year, the chapters are fairly short and diverse enough to be used to suit the needs of students anywhere. They are concise enough to be used either as daily lessons or home assignments, in most instances.

Definitions of new words are at the beginning of each chapter. The main ideas and important terms are italicized with an excellent summary found at the end of each chapter. Stimulating problems and a list of things to do are also included in each unit.

Paragraph headings are in heavy type either as important statements or questions that are answered in interesting detail. Almost every page has a photograph or diagram with very short but pertiment information.

An extensive glossary, index and appendix are contained in the last fifty pages. Tables are on the inside covers.

Only the most important industrial processes are presented and the older theories have been eliminated. It is a superior book, well arranged for both the college preparatory or general chemistry student. It covers all the essential subjects found in most chemistry texts including petroleum and the common organic compounds. Even with its up to date information, it could have included some additional information on the modern theory of ionization with particular reference to the formation of the "hydronium" ion during acid-base reactions.

Teachers considering adopting a new chemistry text would do well to examine this book either for class use or possibly as an addition to their library science shelf, for reference or supplementary reading.

JOHN D. WOOLEVER

REVIEWS AND EXAMINATIONS IN ALGEBRA, Second Edition, by Winfield M. Sides and Oswald Tower of *Phillips Academy*, Andover, Massachusetts. Cloth. Pages iii+183. 14×20 cm. 1953. D. C. Heath & Co., Boston, Massachusetts.

This book is a revision of the first edition by the same authors. Several chapters have been rewritten, and a new chapter—Prize Examinations—has been added.

In the preface to *Reviews and Examinations in Algebra* the authors state that this book is "designed for use in conjunction with standard textbooks in elementary algebra" and that it is "planned also for students who have completed the customary work in this field but who wish to keep the principles of this basic subject alive and to meet the challenge of more difficult problems."

To meet these objectives, the authors have divided their review into two parts: Part 1, a review consisting of two divisions—Algebra to Quadratics and Quadratics and Beyond, and Part 2, Examinations and Supplementary Review Questions.

Part 1 contains a great number of exercises requiring both mechanical manipulation and analysis, progressing from average to very difficult; thus many teachers will find uses for these exercises not only as a review of basic algebra, but also as a challenge to the ability of the more gifted student. Part 1 is comprehensive in its survey of beginning algebra and offers much for second year courses including such review topics as Theory of Quadratic Equations, Variation, Progression, Logarithms, and some trigonometry.

Part 2 of this book is made up entirely of examinations ranging from college entrance examinations to examinations used in secondary schools; e.g., Andover, Exeter, Loomis, Taft, and Hotchkiss. The examinations are geared to individual needs, and are important from many aspects including a good source of preparatory algebraic material, achievement measurement, and preparation for college entrance examinations.

The book, as a whole, is attractive, contains an easily followed table of contents, and includes tables of squares and square roots, trigonometric functions, and four place logarithms.

This reviewer believes *Reviews and Examinations in Algebra* meets its first objective well, and that teachers will find use according to their individual purposes.

LeRoy V. Stoldt
Lyons Township High School
La Grange, Ill.

College Algebra, by Ross H. Bardell and Abraham Spitzbart, *University of Wisconsin*. Cloth. Pages ix+197. 14×21 cm., 1953. Addison-Wesley Publishing Co., Cambridge, Mass. Price $3.50.

The authors of this text have done a good job of selecting and organizing the material for a one semester course in college algebra. By careful arrangement and organization, this material is contained in a book of less than two hundred pages. The students should be able to complete the entire text, rather than study selected chapters from a larger text.

The first forty pages contain a review and extension of high school algebra. The first chapter contains a discussion of the number system of algebra, including positive and negative integers, rational, irrational, and complex numbers. The need for each number system is explained. The number of exercises in these introductory chapters should be adequate for all except the poorly prepared students. The theory is well explained and illustrated by worked examples.

After the introductory chapter, the function concept is introduced and the course organized around it. Thus the chapter on theory of equations follows first and second degree and irrational equations in one unknown. Some of the usual material on the theory of equations is omitted, and equations are solved by the method of interpolation. This seems to be the practical method, especially for those students who do not study advanced mathematics.

The theory of determinants follows the chapter on equations in two and three unknowns. The arrangement of the work on mathematical induction and the binomial theorem furnish additional examples of the careful organization of the text.

The text contains tables of logarithms, powers and roots of numbers, and American experience table of mortality. The graphs and illustrations are well done, and the pages are well arranged and easily read. Schools offering a one semester course in college algebra should consider this text.

Hill Warren
Lyons Twp. Junior College
LaGrange, Ill.

INTERMEDIATE ALGEBRA FOR COLLEGE STUDENTS, Revised Edition, by Thurman S. Peterson, *Portland State Extension Center*. Cloth. Pages viii+369. 14×21 cm. 1954. Harper and Brothers Publishers, New York, New York. Price $3.25.

This is a new edition of a successful text for college students who have had not more than one year of high school algebra. It is a real revision as the exercises and problems as well as the discussions are largely new. Since it is written for students who have not studied algebra for four or more years, it should meet their needs. This is accomplished by a thorough review of elementary algebra with a large number of exercises and a complete discussion of theory. This text provides a review of arithmetic operations, and much practice in the translation of verbal statements to symbols. The student has much experience with algebra before multiplication and division of polynomials are encountered.

Others features which characterize this text are: the definitions and theoretical discussions are in easily understood language, and illustrated by many worked exercises. Fundamental principles and rules are set off in boxes to call attention to them. There are large numbers of well graded exercises with reviews at the ends of the chapters. There are many verbal exercises and the solutions of problems is given much attention. At the ends of the chapters, the author gives problems of special interest to the superior student or which may be used by mathematics clubs. Answers are provided for the odd numbered problems.

This book contains all of the material usually found in intermediate algebra texts, and could be used by high school as well as college classes. Instructors of both groups of students will appreciate the generous supply of exercises and problems, and the easily understood theoretical discussions. It is one of the better intermediate algebra texts.

HILL WARREN

FUNDAMENTALS OF COLLEGE MATHEMATICS, by John C. Brixey and Richard V. Andree, *The University of Oklahoma*, Norman, Oklahoma. Cloth. Pages xiv+609. 16×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $5.90.

There seems to be no agreement among teachers as to the proper content of a single year course in college mathematics; as a result many texts appear to try to agree with a large number of different viewpoints, often to the detriment of the text. To some extent, there seems to be in this text a tendency to range widely and include much—whether this adversely affects the quality of the book is a matter of opinion. As an example, some instructors will feel that a chapter of 25 pages is entirely too brief to devote to as much material in the field of statistics as is discussed; as another example it is doubtful if devoting a half page to the hyperbolic functions is of any real value. This does not necessarily condemn the book. There are many interesting features, for example: drill in algebraic manipulation through use of the delta-process rather than artificial problems; a discussion of nomograms (very brief, however); hints, partial solutions, and graphs rather than mere answers for the odd problems (some instructors may feel too much information is thus provided); interesting pictorial material.

Some features seem to this reviewer very well done. As illustrative examples: the correct statement that certain expressions do not have rational factors, rather than that they cannot be factored; a rigorous statement, using the epsilon-delta notation, of the conditions needed in order that a function has a limit; a footnote pointing out that a negative number does have a logarithm; the pointing out that a polar locus may be symmetric, even though a given test for symmetry fails (however, why not point out an excellent example here of the distinction between a *necessary* and a *sufficient* condition?). Certain other items perhaps show less careful attention to detail: the first sentence on page 152, referring to the mantissa of a logarithm, can be confusing if the student is dealing with a negative logarithm; the inverse secant and cosecant are not defined, nor is it

mentioned that alternative definitions exist; in the discussion of parametric equations it is not pointed out that the rectangular and parametric equations may not be equivalent; on page 414 the student is asked to obtain an "equation of the endgate of a dam" (does this mean the equation of a plane, or of a boundary curve in the plane?); in the discussion of the behavior of loci for large values of $|x|$ certain "non-standard" notations are introduced (even though the customary notations may be misinterpreted, it is often a questionable help to avoid these, for sooner or later the student will encounter them).

Obviously authors and publishers feel any new text is an improvement over existing ones, or the work would not appear. This is no exception. As the authors point out, if one does not wish to use the book in its entirety, selections may be made which emphasize college algebra and statistics, or emphasize analytic geometry, including solid geometry; or which provide a terminal course. Suffice it for this reviewer to say that in spite of this obvious attempt to appeal to many groups, the book is mathematically sound, it contains an ample supply of good problems, and definitely merits consideration as a text in first year college mathematics. Certainly some of the text shows departure from tradition, without a radical throwing away of many topics and methods which have stood the test of time.

CECIL B. READ
University of Wichita

CALCULUS, by George B. Thomas, Jr., *Associate Professor of Mathematics, Massachusetts Institute of Technology.* Cloth. 614 pages. 15×23 cm. 1953. Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass. Price $3.85.

The author's *Calculus and Analytic Geometry* was reviewed in the June, 1952 SCHOOL SCIENCE AND MATHEMATICS; the present book is to a large degree a new edition of that work, so comments will be restricted to changes. The type is easier to read than in the earlier edition, but unfortunately the paper used is so thin that material on the other side shows through. Unfortunately, in some places the use of bold face type for important formulas has been discontinued.

Perhaps 50 to 60 pages of material on analytic geometry do not appear in the *Calculus*. A 30 page chapter on determinants and linear equations has been dropped; a brief discussion of Fourier series has also vanished. A chapter on complex numbers and functions has been omitted, and a chapter of about 35 pages covering elementary differential equations has been added.

There are some minor changes in the problem lists, a few revisions at other points, for example the material on curve plotting is revised and improved. (Material on partial fractions is revised, and in the opinion of the reviewer, inferior.) The later text contains a section on the average value of a function, a discussion of moments as used in statistics, some brief notes on the application of calculus to economics (topics not in the earlier book).

Some instructors will object to graphs of the sine and cosine which use a different scale on the two axes (or, actually, as on p. 124, omit the scale entirely on one axis). The author does not discuss the centroid or center of gravity area, but of a thin homogeneous plate—a point of view some instructors will applaud —but spoils it by asking in problems for the center of gravity of certain areas. This is one of the few books to point out that the Cartesian and parametric equations of a locus are not necessarily co-extensive.

The comments in the earlier review are in general still valid—however, perhaps the author felt he was too enthusiastic. He has omitted his "Hooray!" at the end of his demonstration that a particular method works. Why spoil a nice touch?

CECIL B. READ

A PHILOSOPHICAL STUDY OF THE HUMAN MIND, by Barrell, Joseph, *Professor of Philosophy, Beloit College.* Cloth Pages xii+575. 14×21.5 cm. 1954. Philosophical Library, New York, N. Y. Price $6.00.

The illusory term "mind" has been treated in many ways by psychologists. To some ("He has a good mind.") it becomes synonymous with intelligence, native capacity or performance. To others ("It's on my mind" or "He kept it in mind.") it is a repository of experience. For a reviewer who considers himself a scientist, it is difficult to categorize Barrell's concept of mind except to say that it is not inconsistent with that of personality.

In the first chapter the author makes clear that his purpose in studying the human mind is philosophic in nature, namely, to pass on "the wisdom possessed by one human being which can be communicated to another. In particular, it will mean wisdom about the human mind. . . . " He indicates further that such a philosophic study "will do more than merely collect truths to be gleaned from the different schools of psychology . . . [Rather] such a study will present a certain amalgamation of the truths so gleaned . . . [thus producing a] unified consistent theory [of the mind that will] be of demonstrable use." In attempting to produce such a demonstrable theory the author becomes eclectic and borrows from the "truths" of *S-R* psychology of Thorndike, the type systems that were first debated by the Greeks, the "half-truths" (according to some psychologists) of Freud, and the socially unacceptable behaviors consigned to the study of abnormal psychology.

All in all, using simple anecdotes of day-to-day behavior the author covers topics such as the conscious and unconscious minds, extroversion and introversion, thinking and feeling, emotions, thought analysis, and the relationships among various forms of maladjustments and psychoses. The final chapter is one of consolidation and review—an effort to present a bird's-eye view of the whole of what he admits is a "long and rambling . . . dissertation on the human mind."

In essence, one might find this tome something to read before a fireplace. Without doubt a teacher of psychology would find much in here to use to "pep-up" his teaching, many suggestions that would cause him to say "Why didn't I think of introducing the topic that way before." It is doubtful however whether he would replenish his basic knowledge.

GEORGE GREISEN MALLINSON
Western Michigan College
of Education

EDUCATIONAL PSYCHOLOGY, by Lee J. Cronbach, *Professor of Educational Psychology, University of Illinois*, Cloth. Pages xxvii+628. 16×23 cm. 1954.

One educator was heard to state recently that if all books on educational psychology were laid end to end on the Sahara Desert, it might be a good idea! It is doubtful however whether he had seen this one. With the vast number of publications in this area it would seem that a new entry should cover the same units more effectively or present a new organization. This reviewer believes that this book has succeeded in doing some of both.

The book covers all the traditional areas such as the aims of education, learning, readiness, motivation, emotions and personality. Yet a method of approach generally alien to college textbooks is used, namely introducing the areas of study with anecdotes, examples and case studies. From these introductions the areas are developed interestingly and inductively.

Probably the outstanding features are the format and illustrations. Section and paragraph headings are set off in bold face, illustrations are well done and are integrated with textual materials by means of clever legends, and the type is easy to read.

One may suggest that occasionally the book seems to be "written down," namely that certain passages seem to be patronizing or included for their amusement value. At times also certain presentations tend to be a bit superficial—for example the one dealing with the sociogram on pages 159–161. A person would have to know a great deal about sociograms to "follow" the discussion. Students of educational psychology for whom the book is designed probably would not have such knowledge.

Without doubt this text is going to be a source of concern to its competitors. It is among the very best.

GEORGE GREISEN MALLINSON

PLANE TRIGONOMETRY WITH FOUR-PLACE TABLES, Second Edition, by Arthur W. Weeks, *The Phillips Exeter Academy*, and H. Gray Funkhouser, *The Phillips Exeter Academy*. Cloth. Pages viii+197+37. 14.5×23 cm. 1953. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York 3, N. Y. Price $2.88, without tables $2.68.

The authors have made no fundamental changes in the preparation of this new edition of their text. Some exponential equations have been added to the work in logarithms in the introduction to the book. Cumulative review exercises and a comprehensive final review consisting of modern type questions have been added.

The approach to Trigonometry is through its computational side. The first three chapters carry the student through the solution of oblique triangles by the laws of sines and cosines. Chapter eight contains additional methods of solving triangles. The usual topics of Trigonometry are covered in the remaining chapters. The representation of functions as lines in a unit circle is omitted except for a brief mention in a discussion of the origin of the names sine, cosine, and tangent.

When introducing new materials, attention is focused upon fundamental methods and ideas. A large number of illustrative examples are completely worked out and explained. Typical of this method is the presentation of the evaluation of trigonometric functions of any angle. Good diagrams and complete discussions make clear to the student the values and signs of functions in the various quadrants. Generalized formulas are given later. Attention to fundamental ideas is shown also by the stress given to the fact that the formula $\text{Sin } 2A = 2 \text{ Sin } A \text{ Cos } A$ applies also to $\text{Sin } A = 2 \text{ Sin } \frac{1}{2}A \text{ Cos } \frac{1}{2}A$ and similar situations.

The presentation of graphs of trigonometric functions is extended to graphs of some variations of the sine function and to approximate solutions of equations by graphs. There is an abundance of problem material for every unit in the book. Applications of Trigonometry to mil measures, vectors, and navigation problems in plane, parallel, and middle latitude sailing are included.

Four place tables are given for computational work. Degrees are divided into intervals of 6′ or .1°. The tables list secant and cosecant as well as the usual four functions. A double page is used for each function instead of listing all functions in separate columns on a page. Significant figures are discussed briefly so that students gain some understanding of the interpretation of results of their computation. As in most textbooks, more stress could be given to this topic.

Aside from the omission of a complete discussion of functions as lines in a unit circle, the text provides a very complete course in trigonometry with many varied applications to suit individual tastes.

REINO M. TAKALA
Hinsdale Township High School
Hinsdale, Ill.

MATHEMATICS FOR EVERYDAY LIVING, by Adele Leonhardy, *Chairman, Mathematics Department, Stephens College, Columbia, Missouri*, and Vivian B. Ely, *Head, Mathematics Department, George Washington High School, Indianapolis, Indiana*. Cloth. Pages ix+470. 14.5×22 cm. 1954. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York 3, N. Y. Price $2.96.

This text in general mathematics stresses the mathematics of consumer purchases of goods and services. The authors state that it was written to make the student more informed and competent in dealing with the mathematical aspects of the problems of the consumer, and to provide opportunity for the student to improve the arithmetic skills that are essential in the solution of the problems

of everyday living. Unlike some texts in the field of general mathematics, there is no work in algebraic equations, intuitive geometry, or related fields.

Topics treated include getting more for your money; spending for food, clothing, and shelter; buying on the installment plan; borrowing money; insurance; investments; paying for government services, and spending for recreation. Parts II and III of the text contain a complete review of basic arithmetical operations with whole numbers, fractions, mixed numbers, decimals, and percents.

Materials are presented in an interesting manner with clearly explained sample problems before each set of exercises. There is a wealth of problem material for assignments. Each unit contains a very thorough coverage of materials on the topic being discussed.

Students who study traditional mathematics in high school as well as those who follow the track of general mathematics, would profit from a study of a course such as outlined in this text.

REINO M. TAKALA

NATIONWIDE ANNUAL SEARCH FOR FUTURE SCIENTISTS BEGINS

The Science Talent Search, through which thousands of youths have entered the nation's laboratories for careers in research, was launched when high school students throughout the country were invited to compete in the fourteenth annual competition.

A total of $11,000 in Westinghouse Science Scholarships awaits winners of the Search. They will attend the Science Talent Institute here next spring. These forty finalists will be selected from thousands of entrants from participating public, private and denominational schools. The five-day Institute will feature addresses by some of the nation's leading scientists.

Dr. Leonard Scheele, surgeon general of the United States Public Health Service, who addressed the 1953 Science Talent Institute, in a statement today, commented on the Search as follows: "The range of sciences covered by the young people who embark on the Talent Search is very encouraging. Although the medical sciences are included in the larger group of 'life sciences,' medical research is a focusing of all sciences—life, physical, social—on problems related to the health of man. Even physiology cannot be studied without applying mathematics, physics, and chemistry. Nowadays, medical research is even concerned with the earth sciences and with agricultural sciences—because the kinds and qualities of foods have a direct effect on human health. In my opinion, one of the most valuable aspects of the Science Talent Search is that in a very real sense it touches on every facet of medical research."

The boy or girl, whose all-around performance in the Search as well as personal qualities are judged the most outstanding, will receive a $2,800 Grand Science Scholarship. Other scholarships ranging from $100 to $2000 will be awarded to the remaining 39 finalists during the Institute.

The forty finalists will be invited to Washington to attend the Science Talent Institute beginning February 24. Scholarship winners will be announced at the conclusion of the Institute. In addition, 260 other boys and girls who show "excellent promise of becoming creative scientists" will receive honorable mention citations.

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39% of all classrooms are *overcrowded*

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48% of all elementary classrooms are *overcrowded*

At least 80% of our elementary schools do *not* provide facilities for: kindergarten, music rehearsals, library, gymnasium, auditorium, cafeteria

37% of all elementary classrooms are rated as *unsatisfactory*

—From *NAM News* by the National Association of Manufacturers

[1] On the basis of 30 square feet per pupil; 25 pupils per teacher on elementary level and about 30 on high school level. There are wide variations.

CURTIS — MALLINSON

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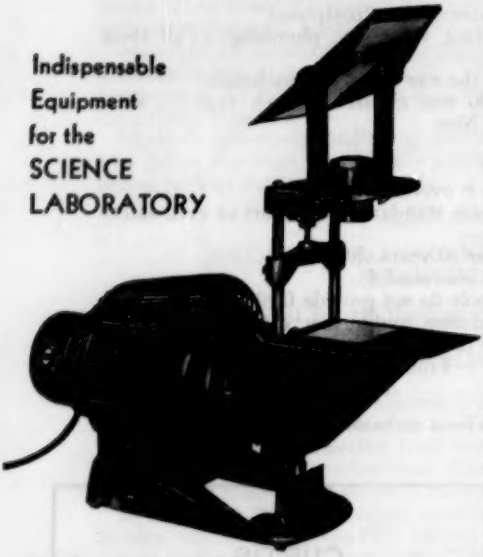

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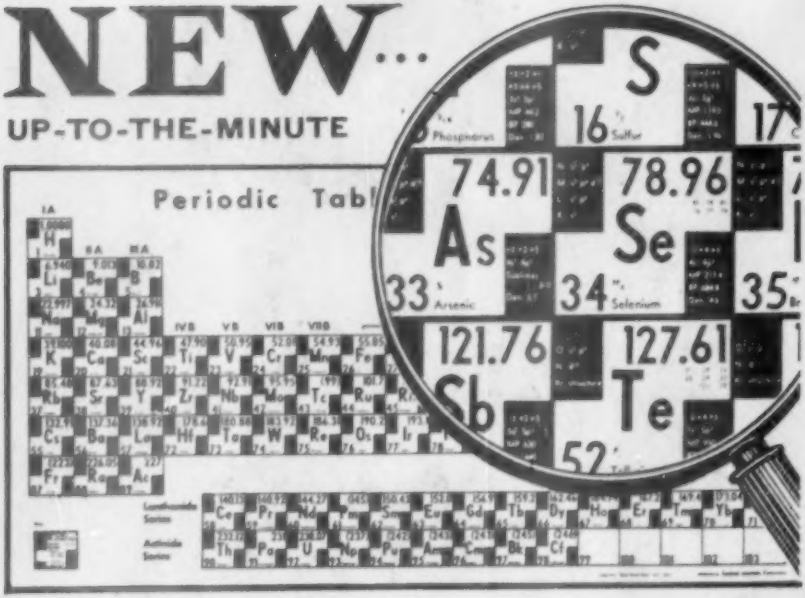

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